The Decomposition of Changes in Average Pay into Job-Mix and Wages-Paid Effects

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The Decomposition of Changes in Average Pay into Wages-Paid and Job-Mix Components

1. The Issue

There have been significant changes in average pay over time in various regions, states, and communities. Figure 1 shows the change in average pay in the Mountain West as well as for the nation as a whole. One common explanation for these changes is that changes in the "job mix" or the industrial structure of employment are largely responsible. We seek to evaluate a particular form of that hypothesis for a particular region, the Mountain West[1], for a particular time period, the 1978-1988 period when average pay declined dramatically and then stabilized. See Figure 1. The particular form that the "job mix" hypothesis has taken in this region is that it was the loss of jobs in the *natural resource* industries or the goods-producing industries during the 1980s that caused the decline in average pay. That is, it was changes in the industrial structure of employment or industrial job mix that caused the decline.

2. The Economic Setting

We assume we are talking about a region that is small enough so that it is strongly linked to the national economy by the free movement of labor, capital, goods, etc. That is, it is an *open* economy embedded in the larger national economy. In particular we assume that national economic forces set the return to different types of productive labor characteristics (pay). Local pay levels, however, may deviate from those national levels due to locally specific characteristics such as amenity qualities that are important to households, productivity or cost characteristics that are important to particular industries, and land rental costs and cost of living that are important to both. These locally specific characteristics (fixed effects and/or non-traded factors, goods, and services) can lead to local compensating pay differentials. In particular we assume that national market forces set the structure of pay among industries and occupations but that the local *level* of pay for all workers may be higher or lower than national levels due to these compensating differentials.[2]

In an open economy, regional output mix is not primarily determined by the shifting of labor from one industry to another, as capital and other inputs are held constant. We are operating in an open economy in which labor, capital, and other inputs can move in and out to take advantage of profitable opportunities with the cost of the inputs largely dictated exogenously by the national economy.

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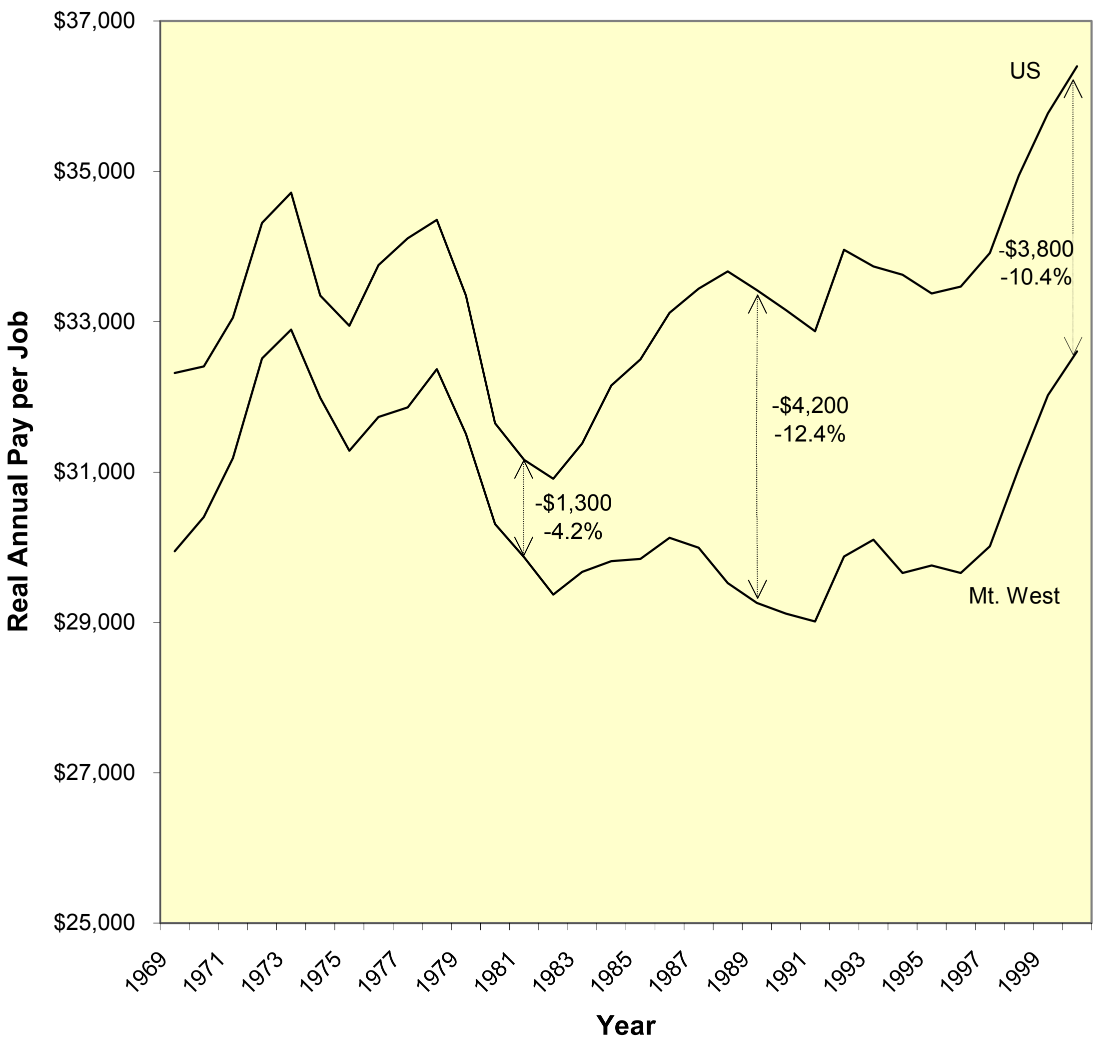
[1] The Mountain West is an eight state region including Montana, Idaho, Wyoming, Utah, Colorado, Nevada, New Mexico, and Arizona. It is to be distinguished from the "Rocky Mountain" region which is significantly smaller since it does not include Nevada, New Mexico, and Arizona.

[2] The implicit assumption here is that all workers make similar evaluations of the amenities found in different locations. Different industries respond to the wage differences that result and make location decisions based on whether those wage differentials are offset by differences in industry productivity at different locations due, for instance, to lower spatial density in high amenity areas.

Figure 1: US and Mountain West Real Pay per Job



This means that within the local economy, average pay in each industry and the share of total employment within that industry are not constrained by either local markets or firms' optimization efforts. As labor *shares* shift among industries, there is no constraint requiring that wages in the sectors with growing shares move downward because of, say, locally diminishing marginal productivity. This is true for several reasons. First, changing employment *shares* do not require that labor actually move from one industry to another. All industries could be adding workers even though some labor shares among those industries were falling and others were rising. An industry whose employment share is declining may actually have expanding employment, just at a slower rate than the overall economy. Between 1978 and 1988 half of the industries in the Mountain West that lost employment share actually expanded their use of labor.

Second, capital and other inputs in the region are not fixed. Expanding sectors do not have to draw resources away from other sectors of the regional economy in absolute terms they can obtain them from the national economy. Third, in particular, the additional labor employed by expanding industries does not have to be “taken” from other sectors. It can come from a growing workforce. In an expanding region like the Mountain West, net in-migration from the larger national economy can provide the new workers that expanding industries seek. An industry whose share of employment is increasing may simply be gaining a larger share of the in-migrating labor force. In the Mountain West between 1980 and 2002 net in-migration was the source of 3.4 million new residents or 30 percent of the 1980 population. Net in-migration was bringing, on average, 155,000 new resident a year to the region.[3] In addition, of course, labor force participation was also rising, increasing the number of workers available locally.

3. The Data Available and Used

Pay per unit of work effort (hourly or weekly wages) is not available for all geographic areas, industries, and types of jobs. This data limitation has often led to the use of average pay per job as a measure of pay. The Bureau of Economic Analysis collects data by two-digit industry down to the county level on total earnings and wage and salary earnings. Data on employment (jobs) is available by two-digit industry down to the state level. This data allows the calculation of average pay per job for each two-digit industry and the state as a whole.

The sources of this average pay data are reports of annual payroll by business firms, reports on the net income from self-employed individuals, and reports of the number of different individuals who earned pay during the year in particular firms. Since a person who holds multiple jobs will be counted as a pay earner by all of her employers, the data only allows a reporting of pay on a *per job* basis, not a per employed person basis.

The earnings data is not reported on an average pay basis. Average pay has to be *calculated* by taking the reported annual total earnings within a particular industry and dividing by the number of reported jobs. That is, an arithmetic average annual pay per job can be calculated by two-digit industry and for the region as a whole. Alternatively, if it were considered appropriate, a geometric average pay per job could also be calculated using this data.

We do not have data on total hours of work effort, the pay per unit of work effort, quantities or cost of other inputs (capital, raw materials, intermediate goods, human capital, etc.). We only have data on two-digit industry total earnings, total jobs, and the industrial distribution of those jobs. From that data we calculate the average annual pay per job, which is the economic statistic that has been the subject of considerable public policy discussion.

[3] US Bureau of the Census, Demographic Components of Population Change. For 1980-1990, only a “residual” population change was available representing the population change that was not explained by births minus deaths. That residual also contained an error component not related to net in-migration.

4. A General Functional Statement of the Decomposition Problem

Let the regional average pay per job that we are trying to relate to changes in industry average pay and job mix (industry employment shares) be indicated by E and the sector version E_i. Let the job mix be described by job shares in each industry, S_i, where $S_i = J_i/\sum J_i$. The J_i are the number of jobs in a particular industry.

We observe that within a region E has changed by the amount $E^1 – E^0$ between the initial and terminal year. We want to relate this change to changes in the E_i and S_i.

(1) $E^1 - E^0 = f(E_1, E_2, \ldots E_n; S_1, S_2, \ldots S_n)$ for all industrial sectors 1….n.

We would like to determine what the impact of just a change in industry wages is, the role of industry labor shares held constant, as well as the impact of just a change in the industry labor shares, the industry wage levels held constant. This can be easily done by simply expressing the regional average pay as a weighted average of the industry average pay using the employment shares as weights and then taking the total differential:

(2) $E^1 – E^0 = \sum s^0_i*(E^1_i – E^0_i) + \sum E^0_i*(s^1_i-s^0_i) + \sum(E^1_i – E^0_i)*(s^1_i – s^0_i)$

The first term is the “wages-paid” effect; the second is “job-mix” effect; and the third is not easily categorized since it involves the interplay of both changes in average pay and changes in job mix. We label this third term the “interaction” effect. Alternatively, if the focus is on the wages-paid effect, the other two terms, the residual, could be combined, as Cooke does, to represent the job-mix effect. In equation (2) we have used the initial year as the base year. We discuss alternatives to that approach below.

Alternatively, following Cooke, one could use a Taylor series expansion of the general functional statement in (1) that average regional pay is a function of the average pay in each industry and industry shares in total employment. If we focus on the “wages-paid” effect, the impact of changes in industry average pay when job mix is held constant, the first two terms in the Taylor expansion would be:

(3) $E^1 – E^0 = \sum f'(E_i^0)*(E_i^1 – E_i^0) + ½ \sum f''(E_i^0)*(E_i^1 – E_i^0)^2$

Where f’ is the vector of partial derivatives of E at the initial point with respect to industry average pay and f’’ is the matrix of second order partial derivatives of E with respect to industry wages.[4]

$f'(Ei^0) = s_i^0$ for average earnings calculated as a arithmetic weighted average.

[4] The presentation of the mathematical relationships here is only suggestive. Because there are a large number of variables (pay and employment shares in each industry), all of these relationships should be presented in vector and matrix form. Cook’s analysis is unconvincing partially because he ignores the fact that both the algebraic and calculus manipulations need to be carried out in matrix form.

$f''(E_i^0) = 0$ since the s_i^0 are being held constant. As a result, there is no second term in the Taylor expansion. Then the wages-paid effect is

(4) $$\sum s_i^0(E_i^1 - E_i^0)$$

which is the same as the first term in expression (2) derived above by taking the total differential of the definition of the weighted average. This is the "wages-paid" effect, the impact of changes in average pay across industries when the job mix is held constant.

If, for some reason, we want to measure "average pay" in terms of the weighted geometric mean, E_G, then

(5) $$E_G = \prod E_i^{si} \quad \text{or} \quad \ln E_G = \sum s_i * \ln E_i$$

The log version of the geometric average is the same as a weighted arithmetic average where the log of average pay replaces average pay. That substitution can be made in equation 2 above to obtain the decomposition of the weighted geometric average into the wages-paid, job-mix, and interaction effects. The wages-paid effect would be

(6) $$\ln E_G^1 - \ln E_G^0 = \sum s_i^0 * (\ln E_i^1 - \ln E_i^0) \quad \text{or} \quad E_G^1/E_G^0 = \prod (E_i^1/E_i^0)^{Si0}$$

when the change in geometric averages is measured as the ratio of the end year to initial year pay and the initial year industrial shares of employment are used as weights in constructing the geometric average.

Simply making use of the definition of an arithmetic (or geometric) weighted average in order to carry out the decomposition makes sense in the setting of an open economy. As pointed out above, in an open economy industry average pay and industry labor shares are not locally constrained. Average pay is heavily influenced by the national labor market and with the free flow of labor into the local economy from the national economy, firms and industries do not have to compete with each other to obtain the labor force they need. In that setting conceptually holding industry labor shares constant while allowing industry average pay to change or holding industry average pay constant while allowing industry labor shares to change as we do in this decomposition does not violate economic constraints linking these two variables.

5. Alternative Specifications of the Decomposition

In the above specification of the wages-paid effect, the industrial structure in the initial year was held constant because that is the hypothesis regularly put forward in the public dialogue: If the industrial structure of employment **had not changed**, regional average pay would not have declined or, at least, would not have declined anywhere near as much.

However, one could carry out the analysis holding industrial structure fixed at the level attained in any other year. In particular, one could use the industrial structure in the end year as the reference point. It is not clear what the relevance of that approach would be: If the industrial structure in 1978 had been the same as the industrial structure that would have developed by 1988, how would average regional pay have changed between 1978 and 1988? Alternatively, one could use the average of the jobs shares in the initial and end year. This ambiguity as to what year weights to use should be familiar from other economic situations. For instance, in evaluating how price levels have changed between two years, one has the choice of using the market basket of goods purchased in the initial year or the market basket of goods purchased in the end year or some combination of the two. Usually, as is the case with hypotheses about the causes of the decline in regional average pay, the question you are trying to answer will dictate the appropriate reference point. If not, averaging across the alternative measures may be appropriate.

The discussion of the decline in average pay is almost always carried out in terms of arithmetic average of pay per job across all jobs, but as pointed out above, it could be discussed in terms of a geometric average. It is not clear what would motivate such an approach. Mathematical simplicity might be a characteristic in some settings since the geometric average would produce an "aggregate pay" function that was linear homogeneous with constant elasticity of the aggregate pay with respect to the component industrial pay. Such a functional form would be similar to those often used for production, cost, and utility functions, but no such "aggregation function" is needed to discuss the issue of average pay in this particular context. However, either type of average can be decomposed into a wages-paid effect, a job-mix effect, and a residual that is associated with the interaction of changes in wages and changes in job mix. It is, of course, important, however, not to mix arithmetic and geometric averages.

8. Empirical Results

a. Bifurcating the Economy into "Good" and "Lousy" Jobs

Decomposition analysis rejects the popular assertions that it was the declines in employment in the goods-producing sectors or expansion in employment in services or declines in employment in high-paying natural resource sectors that cause the decline in average pay in the Mountain West. As shown in the tables below, even if the shares in these popular bifurcations of the economy are frozen at 1978 levels, allowing employment in goods-production and natural resource to significantly expand with the expanding economy and constraining services growth, almost all of the decline in average pay would have taken place anyway. The job-mix effect was in the 0-10 percent range while the wages-paid effect was 90-100 percent. Table 1 shows the wages-paid effect in Montana while Table 2 shows the wages-paid effect for the Mountain West as a whole.

Table1
Decomposition of the Decline in Montana Average Pay into Wages-Paid and Job-Mix Effects

Economic Sectors	1978 Employment Share	1978-1988 Change in Employment Share	1978 Pay per Job	1978-1988 Change in Pay per Job	Percent of Loss in Pay per Job with No Employment Shift
Sector Composition: Non-Farm Goods and the Rest of the Economy					
Non-Farm Goods	18.3%	-3.7%	$39,203	-$9,648	
Rest of Economy	81.7%	3.7%	$24,670	-$3,525	
Total Economy	100.0%	0.0%	$27,331	-$4,956	**94%**
Sector Composition: Services and the Rest of the Economy					
Services	21.1%	5.5%	$21,061	-$2,100	
Rest of Economy	78.9%	-5.5%	$28,733	-$7,053	
Total Economy	100.0%	0.0%	$27,114	-$6,157	**98%**
Sector Composition: Declining High Wage Sectors and the Rest of the Economy					
Declining High Wage Sectors	8.6%	-4.1%	$44,883	-$8,943	
Rest of Economy	91.4%	4.1%	$25,439	-$5,480	
Total Economy	100.0%	0.0%	$27,114	-$6,428	**90%**

The decline in the share of employment in the natural resource sectors of the Montana and Mountain West economy explains almost none (6 percent or less) of the decline in real pay in the region. Rather than shifts in job share from natural resource industries to other sectors, it was an overall decline in real pay across most sectors of the economy, including the natural resource sectors, that explains falling pay. Public discussions of changes in the regional economy have some times focused on other bifurcations of the economy. For example, some have been concerned not just about the shift away from natural resource sectors but the larger shift of employment from all goods-production to services-production. Others have focused on a broader set of high wage industries including, for instance, railroad employment, in which employment has declined while jobs have expanded in much lower-paying industries. Analysis of these types of changes in the industrial structure of employment reveals that the impact of changes in job mix is somewhat larger than the analysis of the natural resource industry bifrucation, but the impact of change is job mix is still quite small, 10-13 percent of the total decline in average pay. Job shifts between these popular bifurcations of the regional economy into good jobs and lousy jobs do not explain the vast majority of the decline in real pay in the region between 1978 and 1988.

Table 2
The Decomposition of the Decline in Mountain West Average Pay into Wages-Paid and Job-Mix Effects

Economic Sectors	1978 Employment Share	1978-1988 Change in Employment Share	1978 Pay per Job	1978-1988 Change in Pay per Job	Percent of Loss in Pay per Job with No Employment Shift
Sector Composition: Natural Resources and Rest of the Economy					
Natural Resources	8.0%	-2.7%	$35,578	-$5,513	
Rest of Economy	92.0%	2.7%	$28,133	-$2,316	
Total Economy	100.0%	0.0%	$28,732	-$2,689	**96%**
Sector Composition: Goods Producing and Non-goods Producing					
Goods Producing	24.2%	-4.1%	$36,832	-$4,005	
Non-goods Producing	75.8%	4.1%	$26,146	-$1,815	
Total Economy	100.0%	0.0%	$28,732	-$2,689	**87%**

b. Decomposing Average Pay Using Sixty-six Two-Digit Industrial Composition

If we move away from these popular bifurcations of jobs and look instead at all of the different ways in which jobs shares shifted among industries, the industrial structure of employment (or job mix) plays a somewhat more important role. For the Mountain West, about two-thirds of the decline in average pay is tied to declines in average pay across industries (the wages-paid effect). However, these employment shifts did not fit the popular characterization of relative job loss in the natural resource sectors or the shift from goods to services. Instead, the somewhat larger impact of changes in job mix in explaining the decline in real pay was associated with shifts in employment *within* services, mining, manufacturing or transportation, not between these sectors. In any case, even in this more disaggregated analysis, changes in industrial structure (job-mix) remained a much weaker force, only half as important as overall declines in average pay (wages-paid effect) in causing the decline in regional average pay.

c. Functional Form and Reference Year Do Not Appear to Matter

In our original decomposition of the decline in real pay, we in effect took the total differential of the statistic in question, average real pay. That statistic is a weighted arithmetic average of pay levels in the various industrial sectors with employment shares are the weights. Since this *is* the economic statistic that is so often discussed, that seemed to be the most direct way to analyze the claims made about the forces acting on it. Abstracted from this public economic dialogue, however, it is reasonable to ask whether the quantitative importance of changes in job mix would be different if average pay were calculated differently and a different decomposition method were used. For instance a geometric average might be used and decomposition carried out using Diewert-type index numbers.[5] We have done the decomposition analysis using

[5] This is the approach taken by Stephen Cooke in his paper prepared for this session. Diewert, W. E., “Exact and Superlative Index Numbers” Journal of Econometrics 4:115-45, 1976.

various alternative functional forms for the decomposition and have found that for the Mountain West the results do not differ significantly from those obtained from our decomposition of the weighted arithmetic average as long as a true, internally consistent, decomposition approach is used. See Table A below.

Table A

The Relative Importance of the Wages-Paid Effect Mountain West, 66 Industry Disaggregation			
Index Used	**Decomposition Method**	**Year Used for Job Mix Weights**	**% of Change in Pay Due to Wages-Paid Effect**
Power/Barrett	**Wt. Arithm Avg**	**Initial Year**	**64.4%**
"Leontief"	**In of ratios**	**Initial Year**	**63.4%**
"CES"	**Wt. Geom. Avg**	**Initial Year**	**65.4%**
Power/Barrett	**Wt. Arithm Avg**	**End Year**	**66.1%**
"Leontief"	**In of ratios**	**End Year**	**66.9%**
"CES"	**Wt. Geom. Avg**	**End Year**	**68.3%**

Since the purpose of our decomposition was to answer the question of what would have happened to average pay in the region if the industrial structure of employment had not changed during the 1980s, it was appropriate to carry out that decomposition by holding industrial job shares constant at 1978 levels. However, in some abstract sense, one could instead use job shares at the end of the period, 1988, or use the average of job shares at the beginning and end of the period. Index number theory tells us that, in general, there is no correct reference period for the weights although the question being asked may provide some guidance. It is possible, then, that if we had used end-of-period weights, we would have found that the role of job mix would be greater than our decomposition showed. That however, does not turn out to be the case. Using either 1978 or 1988 jobs shares or an average of the two does not significantly increase the role of job mix in explaining the decline in regional real pay.

Much of the public discussion pay levels in the Mountain West, however, has not been focused on the decline in *real* pay per job. This may be true in part because people do not think in terms of inflation free dollars and are not aware of how pay has changed in those terms. But more importantly, regional average pay is most often discussed relative to pay in the nation as a whole and relative to other states, rather than relative to prices. The fact that three of the eight Mountain West states (Montana, Idaho, and Wyoming) are in the poorest ten states in terms of pay per job, and two more (New Mexico and Utah) are in the poorest fifteen[6], alarms the region's residents, as does the fact that between 1980 and 1988 the gap between Mountain West and national average pay almost tripled, rising from 4.2 to 12.3 percent. Because people often judge their economic well being relative to that of their neighbors, this focus on the *relative* pay gap is not surprising.

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[6] 2001 data on annual pay per job calculated by dividing total earnings by total employment as reported in the BEA REIS data base.

We have analyze the growth in the *relative* gap to see what portion of this growth is attributable to the changes in the industrial mix of employment, as opposed to changes in pay across all industries (the wages paid effect).

To decompose the real pay gap between the Mountain West and the nation, we focus on the differences between the Mountain West and the rest of the nation in industrial earnings per job and job shares at a point in time, rather that changes over time. The relative pay gap in any given year is $E^{mw} – E^{us}$, where E^{mw} and E^{us} are regional and national average earnings per job in that year. The decomposition of the *relative* pay gap is shown in (4).

$$(4)\ E^{mw} – E^{us} = \Sigma_i\ S_i^{mw} * (E_i^{mw}-E_{i\,,}^{us}) + \Sigma_i\ E_i^{mw} * (S_i^{mw}-S_i^{us})\ - \Sigma_i\,(S_j^{mw}-S_i^{us})*(E_i^{ms}-E_i^{us})$$

where E_i^{mw} and E_i^{us} are earnings per job and S_i^{mw}, and S_i^{us} are job shares in industry i in the Mountain West and the country as a whole respectively in a particular year. We define the first term on the right hand side of (4) as the wages-paid effect, i.e. the portion of the wage gap attributable to differences in earnings per job between the Mountain West and the nation, holding industry employment shares constant at their Mountain West values; the remaining two terms combined, which show the residual gap after accounting for the wages-paid effect, are treated as the job-mix effect.

In order to see how these different components of the relative pay gap between the region and the US have changed over time, the decomposition was calculated for each year from 1978 through 2000. Figure 2 below shows the relative role of the wages paid and job mix effects since 1978 as the real pay gap between the nation and the Mountain West grew.[7]

As can be seen in Figure 2 the relative pay gap is largely due to the wages paid effect, not effects tied to differences in the mix of jobs among industries between the Mountain West and the nation. In 1988, for instance, 83 percent of the gap is due to the wages paid effect and only 17 percent due to the regional job mix. Averaged over the 1978-2000 period, the job mix effect explained only 14 percent of the pay gap between the Mountain West and the region.[8]

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[7] This decomposition is calculated for a 75 industry disaggregation of employment and earnings for the eight state Mountain West region. To perform this disaggregation it was necessary to estimate a small number of undisclosed earnings and/or employment values for specific industries, states and years. The results of decompositions using this 75 industry disaggregation and the 66 industry disaggregation referred to above are nearly indentical.

[8] We are following Cooke in stating the results in this way: The wages paid effect is directly calculated and the remaining pay gap, the residual, is labeled the job mix effect. This definition of the job mix effect combines a term in the decomposition [$\Sigma_i(\Delta S_{j,i}Wmw_{j,i})$ in (4)] that is solely a job mix effect, with an interaction term that involves both differences in job mix and differences in pay [- $\Sigma_i(\Delta S_j,i\Delta W_{j,i})$ in 4]. In this case, the interaction term is significant and negative. As a result, when it is combined with the directly calculated job mix effect, the combined value of the two terms is reduced and the residual job mix effect is quite small. Were we to define $\Sigma_i(\Delta S_{j,i}Wmw_{j,i})$ as the industry mix effect and calculate the wages paid

9. Conclusions

The popular hypothesis that it was changes in the job mix (the industrial structure of employment) that was primarily responsible for the decline in average pay in the Mountain West is not supported by the data. In its usual form in which the economy is bifurcated into “good” and “lousy” jobs, changes in job mix played almost no role in the decline in average pay. It was declines in real pay in both “good” and “lousy” jobs (the wages-paid effect) that explain the decline in average pay.

If the structure of employment is more finely disaggregated, the role of changes in job-mix increases significantly but still explains only about one-third of the change. That is, the job-mix effect remains only about half as important as overall declines in pay across most industries. The job mix changes that matter are changes *within* broad industries such as mining, transportation, manufacturing, and services, not jobs shifts between these broad industries. These two results also challenge the popular job-mix hypothesis.

Changes in the functional form that is use in the decomposition of changes in average pay into job-mix and wages-paid effect and changes in the what year is used as the reference point for the weights does not significantly affect the relative importance of the two effects in the Mountain West states.

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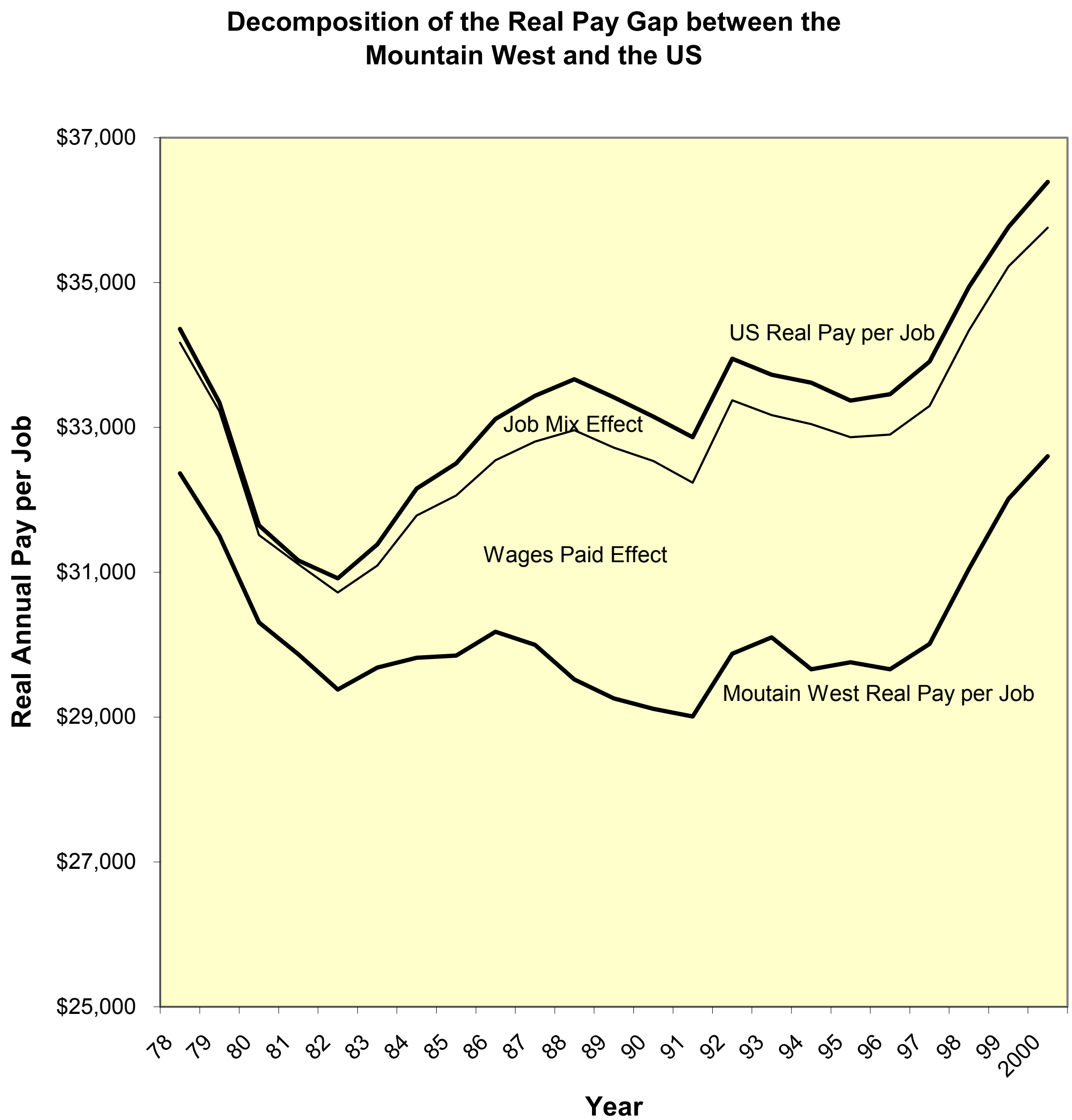
effect as the residual wage gap, the industry mix effect would account for 34 percent of the wage gap and the wages paid effect 66 percent, rather than the 86 percent reported above. The reason for this discrepancy is that differences between the region and the nation in pay by industry and the relative importance of each industry as a source of employment interact in a way that has an impact on the pay gap that cannot obviously be classified as either a job mix or a wages paid effect. On average this interaction term was negative and 20 percent of the total pay gap. When combined with the 34 percent direct job mix effect, the residual job mix effect is the 14 percent reported above.

Figure 2



Decomposition of the Real Pay Gap between the
Mountain West and the US
$37,000
$35,000
$33,000
$31,000
$29,000
$27,000
$25,000
Real Annual Pay per Job
US Real Pay per Job
Job Mix Effect
Wages Paid Effect
Moutain West Real Pay per Job
78
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80
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98
99
2000
Year

Appendix

A Critique of Cooke's Approach to Decomposing Average Pay

a. Introduction

Diewert and others have shown that in the context of production, cost, consumption, or utility optimization, one has to be careful how one constructs indices of changes in output, cost, consumption, or utility or the index of price weighted quantities or quantity weighted prices may be inconsistent with the usual assumptions we make about the characteristics of the underlying functions and the constraints imposed by optimization. In particular, if the underlying, say, production, function is assumed to be a arbitrary twice-differentiable linear homogeneous function, then an index number that accurately reflects changes in output has to be a translog function whose exponents are the average shares of inputs in total factor costs for the beginning and end years. The same is true for a cost function. (Diewert 1976, equations 2.12 and 2.15)

For a production function:

$Q^1/Q^0 = \prod (x_i^1/x_i^0)^{½(si0+si1)}$ (Diewert 1976, equation 2.12)

where Q is the quantity of output in dollar terms, x_i are the expenditures on factor inputs to the production process, and s_i are the shares of input i in the total cost of inputs.

For a cost function:

$C^1/C^0 = \prod (p_i^1/p_i^0)^{½(si0+si1)}$ (Diewert 1976, equation 2.15)

where C is the total cost of production, p_i is the unit cost of factor input i, and the s_i are the shares of cost of factor i in total input costs.

b. Cooke's Approach

These mathematical results are used by Cooke to specify how changes in average pay across a region should be functionally expressed so that the impact of changes in the industrial structure of employment can be separated from the impact of changes in average pay across regional industries. Cook reasons by analogy assuming that for the problem at hand we have an average pay function (an "earnings function"):

$$E^1/E^0 = \prod (E_i^1/E_i^0)^{½(si0+si1)}$$

where E is average pay per job calculated by dividing total earnings by total jobs. The E_i are average pay per job in the disaggregated industries also calculated by dividing total earnings by total jobs in that industry. Note the similarity to the Diewert production and cost functional forms.

Cooke has defined the s_i in two quite different ways. In the November 19th version of his paper he defined them as the share on total regional *employment* found in a particular industry. In the February 22nd version of the paper that was presented at the Western Regional Science Association meetings, he defined the s_i as the share of a particular industry in total regional *earnings*, justifying this by analogy with the Diewert approach to production and cost functions.

c. The Error in Cooke's Approach

Diewert and others were focused on the quantities of inputs into an optimized production or consumption process and the prices associated with them. The relationships between input prices and quantities and the shares in total expenditures that result from optimizing subject to a constraint were central to the derivation of the particular functional form used to measure the changes over time.

Cooke has not defined either a "production" or "cost" function that relates factor *input* quantities and prices to an aggregate value of production or total cost. Instead he is analyzing the way that average pay in regional industries contribute to the aggregate average pay across all industries in the region. He is not analyzing the relationship between different types of labor and the wages associated with them in the context of a firm's or industry's production or cost function. Instead he simply has an aggregate measure of regional average pay and is seeking to relate it to average pay at the industry level within that region. That is, he is simply trying to relate an average value to its component values. The functional form relating these is simply the definition of the average being used (e.g. weighted arithmetic, weighted geometric, etc.). Cooke actually recognizes this and uses the definition of a weighted arithmetic average in his "derivation."

Equally important, Cooke has not defined an optimization problem that would allow him to make the substitutions that Diewert and others made. In particular, Euler's Theorem (or, as Cooke labels it, Hotelling's Lemma) does *not* imply, in this case, that the "shares" refer to shares in total earnings (i.e. factor shares in total costs). The setting we are analyzing is not one where different types of labor are being combined with other inputs to produce a given level of output. We have separate industries producing different products and are only focused on one of each industry's inputs, the total number of employees. The shares in this setting, as the definition of the averages makes clear, are the shares in total employment.

As Cooke's "derivation" makes clear, he does not define an "earnings" or average pay function. Instead he simply introduces the definition of average pay (total earnings divided by total jobs). To this he attempts to apply a Taylor series expansion. However, since average pay is a linear function of industry labor shares and industry average pay, there is no second order term to make such an expansion meaningful. Cook, despite the linear definition of "average pay" assumes it is a quadratic function.

Cooke ignores this characteristic of a linear function and assumes that changes in an industry's average pay will have an impact on the industry's share of total regional employment, even when the analysis is explicitly holding industry shares constant.

Cooke avoids this obvious linear relationship associated with the definition of average pay by positing a "quadratic earnings function in which the overall region's average earnings are a function of the region's average wages in each sector for a given industry structure…The curvature in the earnings function is the result of *the substitution of labor across sectors as relative wages change*. As more labor moves into the high wage sectors the marginal productivity of labor decreases, wages decrease, and overall average earnings increase more slowly." (p. 9, emphasis added)

In an open economy, this simply is not the case. Cooke's analysis implicitly assumes that within the region capital, other non-labor inputs, and other labor are fixed and cannot increase. But in an open economy, capital, labor, and other inputs can freely move into the region. In particular, an industry's share of total regional labor can increase without bidding any labor away from other industries. It can simply draw more heavily on the in-migration of more workers. Furthermore, in an open economy average pay is not determined endogenously within the region. The relative levels of pay for different types of labor are dictated by the larger national economy while compensating differentials associated with local cost of living and amenities lead to local deviations in overall pay levels from those national levels.

In this setting, the use of labor in one regional industry is not related to the use of labor in another. Each industry can attract all of the labor it needs as long as it pays a wage that reflects national wages discounted for local amenities. Within an open local economy there is no "earnings function" linking industrial average pay and the industry employment shares.

In that setting a weighted arithmetic average of industrial average pay, using employment shares as weights, is a consistent index for average regional pay. No other functional form is needed or appropriate. Of course, if by "average pay" the geometric average is meant, then the definition of a weighted geometric average where the weights again are industry labor shares of total regional employment can be used as the index. These two definitions of "average," however should not be mixed as Cook does, since that can lead to an inconsistent decomposition.

d. The Inappropriateness of Using Earnings Shares As Weights

Cooke in his February 2003 WRSA paper claimed that the mathematics of analyzing the wages-paid effect requires that the shares that serve as the industrial weights be measured not as industrial shares of total employment but industrial earnings as a share of total regional earnings. This is incorrect and contradictory for several reasons.

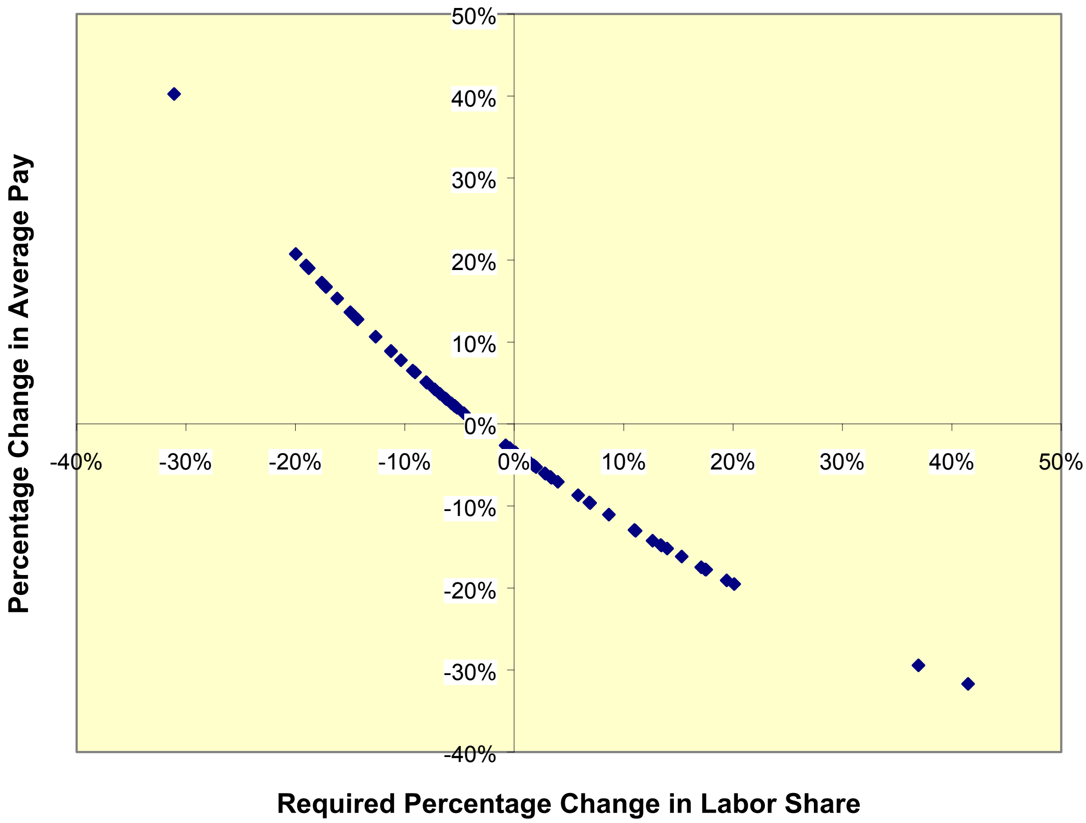
i. Using such weights means that job mix is no longer the focus of the analysis. Since the determination of the impact of holding job mix constant on the evolution of regional average pay was the motivation for this analysis to begin with, using earning shares and holding them constant, whatever it might tell us, does not deal with the problem originally posed.

ii. If industrial shares of total earnings are held constant and average industrial pay is allowed to change, the industrial structure of employment (job mix) *must* change in a way that offsets the changes in industrial pay. See the figure below. Thus, holding earnings shares constant prevents job mix from being held constant. That means that what is calculated cannot be labeled the wages-paid effect (since job mix has been allowed to change) and that the residual cannot be labeled a "job-mix effect" either since some of the job-mix effect was captured in the original calculation.

iii. There is no theoretical or mathematical reason to hold earnings shares constant rather than job shares. Cooke uses an analogy with an optimization problem in the context of a Cobb-Douglas production function to come to the conclusion that earnings shares should be the weights in his wages-paid effect. In competitive factor markets where the production function is Cobb-Douglas, the payments to factors (e.g. wages) are equal to the value of the marginal product and the factor share in the value of total output is equal to the coefficient (exponent) on that factor in the production function. But we are not dealing with a production function here; we are dealing with some measure of average earnings. If that measure is a weighted geometric average pay, then we have a functional form identical to a Cobb-Douglas production function in which the exponents (industrial labor shares) sum to one. The same "adding up" characteristic is associated with this functional form when the weights are the shares in total employment. There is no reason to turn to earnings shares.

In short, if industrial earnings shares are used as weights in this decomposition of changes in average pay, the result is **<u>not</u>** part of any formal decomposition of average pay. It isolates neither a wages-paid nor a job-mix effect.

Relationship between Changes in Average Pay and Changes in Implicit Labor Shares when Earnings Shares Are Held Constant



e. Inconsistencies in Mixing Arithmetic and Geometric Averages

Although Cooke seeks to provide an elaborate mathematical “derivation” of his “Cooke Index” of the wages-paid effect, what he ultimately uses is a simple weighted geometric average of industrial average pay where the weights are the average of the beginning and end year labor shares:

$$\prod (E_i^1/E_i^0)^{\frac{1}{2}(si0+si1)}$$

He then compares this *geometric* measure of the wages-paid effect to the total change (expressed as the ratio of natural logs) in the weighted *arithmetic* average pay:

$$\ln \left(\sum s_i^1 * E_i^1 / \sum s_i^0 * E_i^0 \right).$$

The difference between these two Cooke labels the job-mix effect.

But this mixture of geometric averages and arithmetic averages introduces in inconsistency. A geometric mean is always smaller than an arithmetic mean (unless the elements are all equal). For this reason, there is always a difference between them that has nothing to do with any particular modeling effort. In this case Cooke is attributing to the job-mix effect the difference that results from mixing the two different definitions of the average.

Measuring the job-mix effect as the differences between his geometric definition of the wages-paid effect and the actual change in the weighted geometric average pay could have easily solved this problem. A simple example will demonstrate this inconsistency and its solution.

Table A

An Example of the Internal Inconsistency of the Cooke Decomposition Method

Industry	Percentage Labor Share		Average Pay per Job	
	Initial Year	End Year	Initial Year	End Year
Industry 1	0.50	0.50	$10.00	$10.05
Industry 2	0.40	0.40	$2.00	$2.01
Industry 3	0.10	0.10	$40.00	$50.00

Decomposition of the Change in Average Pay	Cooke Index A Mix of Geom. and Arithm. Avg.	Consistent Use of Averages Weighted Geometric Average	Consistent Use of Averages Weighted Arithmetic Average
Total Change	9.8%	2.7%	10.5%
Wages-Paid Effect	27%	100%	100%
Job-Mix Effect	73%	0%	0%

In the above example, the industrial structure of employment (job mix) does not change during the analysis period, but average pay does. In that situation, <u>all</u> of the change in average pay should be classified as due to a wages-paid effect and none to a job-mix effect. Yet the “Cooke Index” classifies 73 percent of the change as a job-mix effect. If, instead, he had made consistent use of one definition of the average or the other, as Power and Barrett did, all of the change in average pay would have been classified correctly as a wages-paid effect. Cooke, in effect, is labeling the difference between an arithmetic and geometric average as a “job-mix” effect. That is clearly an error.