The denominator in equation 12.29 can be zero in parts of the k_x, k_y plane for certain orientations of $\hat{\mathbf{m}}$ and $\hat{\mathbf{f}}$, in which case $\mathcal{F}[\psi_{\rm t}]$ is not defined. If $\hat{\mathbf{m}}$ and $\hat{\mathbf{f}}$ are nearly parallel to these singular directions, $\mathcal{F}[\psi_{\rm t}]$ can reach high amplitudes in parts of the k_x, k_y plane, and the filtering operation of equation 12.28 may be unstable. Some of these situations will be discussed in Section 12.3.1. Moreover, the imaginary part of equation 12.29 is discontinuous through the origin of the k_x, k_y plane, and this discontinuity can affect the long-wavelength parts of ΔT (Kis [147]).

The steps involved in applying equation 12.28 are (1) Fourier transform the measured ΔT, (2) multiply by $\mathcal{F}[\psi_{\rm t}]$, and (3) inverse Fourier transform the product. Subroutine B.26 in Appendix B shows the implementation of this three-step procedure.

12.3.1 Reduction to the Pole

Positive gravity anomalies tend to be located over mass concentrations, but the same is not necessarily true for magnetic anomalies when the magnetization and ambient field are not both directed vertically. Unless $\hat{\mathbf{m}}$ and $\hat{\mathbf{f}}$ are both vertical, $\Theta_{\rm m}$ and $\Theta_{\rm f}$ will contribute a phase to the magnetic anomaly, which can shift the anomaly laterally, distort its shape, and even change its sign (Figure 12.7).

Exercise 12.2 What directions for $\hat{\mathbf{m}}$ and $\hat{\mathbf{f}}$ cause the phase contribution of $\Theta_{\rm m}\Theta_{\rm f}$ to be -1?

In general terms, if the magnetization and ambient field are not vertical, a symmetrical distribution of magnetization (such as a uniformly magnetized sphere) will produce a "skewed" rather than a symmetrical magnetic anomaly (see Figure 4.9 for several extreme examples).

This added complexity can be eliminated from a magnetic survey by using equations 12.28 and 12.29. If we require $\hat{\mathbf{m}}' = \hat{\mathbf{f}}' = (0, 0, 1)$ in

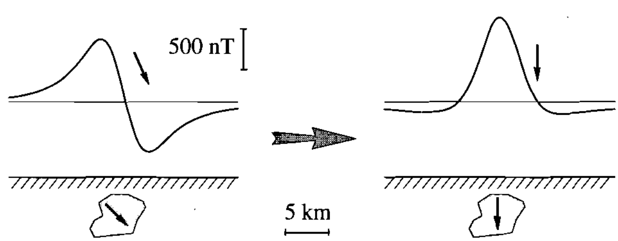

Fig. 12.7. A magnetic anomaly before and after being reduced to the pole.

ation 12.29, then equation 12.28 will transform a measured total-l anomaly into the vertical component of the field caused by the same rce distribution magnetized in the vertical direction. The transformed maly in the Fourier domain is given by

$$\mathcal{F}[\Delta T_r] = \mathcal{F}[\psi_r]\,\mathcal{F}[\Delta T], \tag{12.30}$$

re

$$[\psi_r] = \frac{1}{\Theta_m \Theta_f}$$

$$= \frac{|k|^2}{a_1 k_x^2 + a_2 k_y^2 + a_3 k_x k_y + i|k|(b_1 k_x + b_2 k_y)}, \quad |k| \neq 0, \tag{12.31}$$

$$a_1 = \hat{m}_z \hat{f}_z - \hat{m}_x \hat{f}_x,$$

$$a_2 = \hat{m}_z \hat{f}_z - \hat{m}_y \hat{f}_y,$$

$$a_3 = -\hat{m}_y \hat{f}_x - \hat{m}_x \hat{f}_y,$$

$$b_1 = \hat{m}_x \hat{f}_z + \hat{m}_z \hat{f}_x,$$

$$b_2 = \hat{m}_y \hat{f}_z + \hat{m}_z \hat{f}_y.$$

The application of $\mathcal{F}[\psi_r]$ is called *reduction to the pole* (Baranov and udy [10]) because ΔT_r is the anomaly that would be measured at the th magnetic pole, where induced magnetization and ambient field h would be directed vertically down (Figure 12.7). Reduction to the e removes one level of complexity from the interpretive process: It fts anomalies laterally to be located over their respective sources and ers their shape so that symmetrical sources cause symmetrical anoma-. The direction of magnetization and ambient field are required in uation 12.31, but no other assumptions about the distribution of mag-ization are necessary, except those concerning the lateral dimensions the sources described earlier.

Many of the comments concerning Θ_m and Θ_f in Section 11.2.1 apply $\mathcal{F}[\psi_r]$. The filter attains a constant value along any ray projected from origin of the k_x, k_y plane. Two rays directed in opposite directions complex conjugates of one another, so the imaginary part of $\mathcal{F}[\psi_r]$ discontinuous at the origin. Each ray, in general, differs in value from ghboring rays, but the average of $\mathcal{F}[\psi_r]$ around any circle concentric out the origin is independent of the radius of the circle. Thus reduction the pole has no effect on the shape of the radially averaged spectrum.

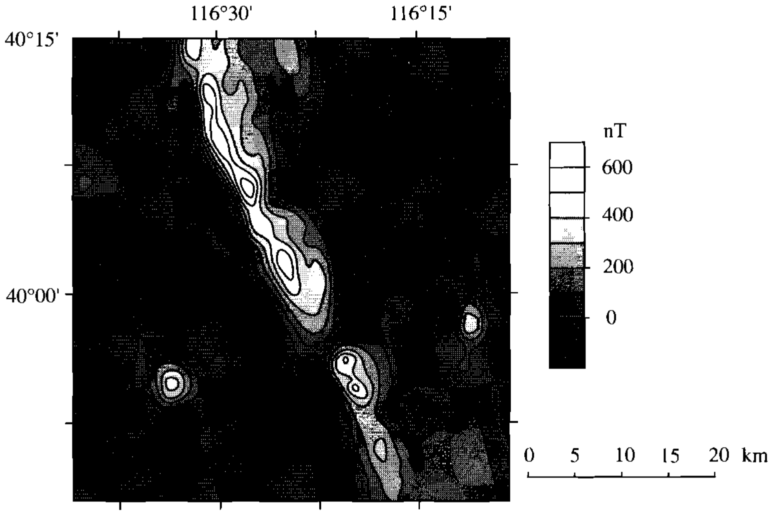

Fig. 12.8. The total-field anomaly of Figure 12.1 reduced to the pole.

Exercise 12.3 Demonstrate the validity of each of these points using equation 12.31.

Figure 12.8 shows the total-field anomaly from central Nevada (Figure 12.1(a)) reduced to the pole. Notice that the transformed anomalies are generally more symmetric than their original counterparts. In particular, the isolated anomaly in the southwestern part of the map is more centrally located over its apparent source, a Tertiary granitic pluton (Figure 12.1(b)). Subroutine B.26 in Appendix B can be used for reduction to the pole by specifying vertical inclinations for magnetization and ambient field.

Problems at Low Latitudes

Figure 12.9 shows two examples of the amplitude and phase of $\mathcal{F}[\psi_r]$: Figure 12.9(a) assumes an inclination of 60° for both the ambient field and magnetization, typical of a mid-latitude survey, whereas Figure 12.9(b) assumes an inclination of 10°, as might be found near the magnetic equator. Both amplitude spectra in Figure 12.9 vary smoothly throughout the k_x, k_y plane (except at the origin), but the amplitude spectrum in Figure 12.9(b) attains large values within narrow pie-shaped segments. Moreover, these high-amplitude segments extend to the longest

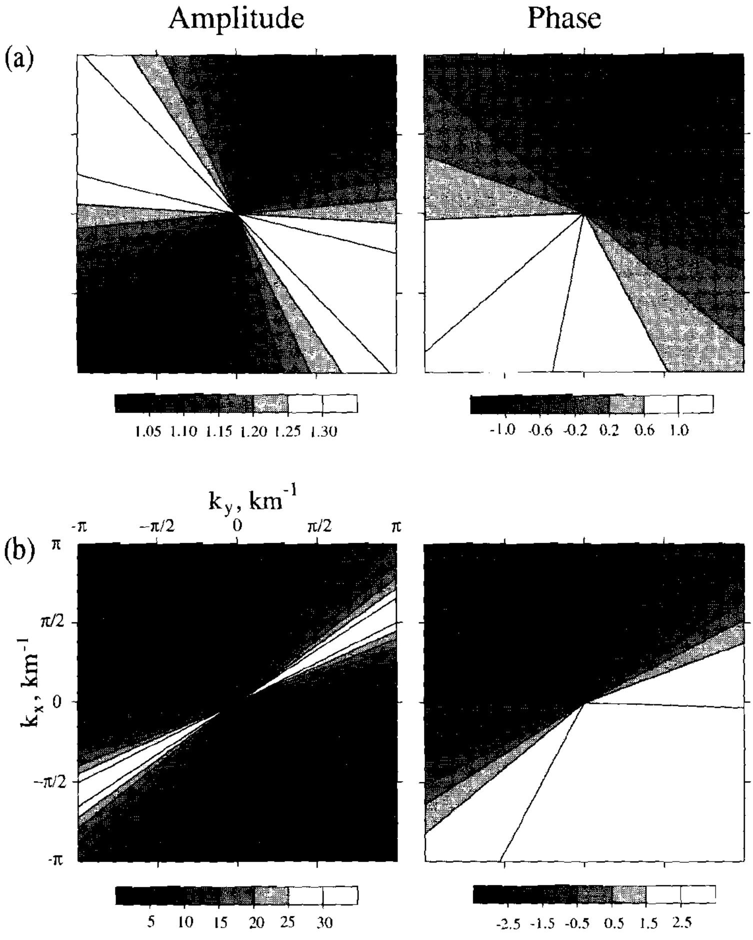

Fig. 12.9. Amplitude and phase spectra for two reduction-to-pole filters. (a) Mid-latitude case, with inclination 60°, declination 30°; (b) low-latitude case, with inclination 10°, declination −30°.

wavenumbers (shortest wavelengths) of the spectrum. This is a characteristic of $\mathcal{F}[\psi_r]$ in low-latitude situations. In applying equation 12.30 in such situations, the measured total-field anomaly, and any noise included within the measurements, will experience this high, directionally selective amplification. The result can appear as short-wavelength artifacts elongated in the direction of the declination.