Knowing the direction of the ambient field, therefore, is sufficient to transform a total-field anomaly into the vertical component of the magnetic field.

It should be clear that such transformations will be limited for certain ambient-field directions in the same way that reduction to the pole is limited at low latitudes. Consider, for example, a total-field anomaly measured near the magnetic equator and caused by a body with purely induced magnetization. Both $\hat{\mathbf{f}}$ and $\hat{\mathbf{m}}$ will have shallow inclinations in this case, and transforming the total-field anomaly into the vertical component of the magnetic field can be expected to be an unstable operation. Any noise within the measurements will generate artifacts, typically short in wavelength and elongated parallel to the declination.

Exercise 12.5 Derive Fourier-domain filters that will transform a total-field anomaly into the horizontal-north component and the horizontal-east component of the magnetic field. Discuss the conditions under which these filters are expected to be unstable.

12.4 Pseudogravity Transformation

Poisson's relation was discussed at some length in Section 5.4, where it was shown that the magnetic potential V and gravitational potential U caused by a uniformly dense and uniformly magnetized body are related by a directional derivative, that is,

$$V = -\frac{C_{\rm m}}{\gamma}\frac{M}{\rho}\hat{\mathbf{m}} \cdot \nabla_P U$$

$$= -\frac{C_{\rm m}}{\gamma}\frac{M}{\rho} g_{\rm m} , \qquad (12.43)$$

where ρ is the density, M is the intensity of magnetization, $\hat{\mathbf{m}}$ is the direction of magnetization, and $g_{\rm m}$ is the component of the gravity field in the direction of magnetization $\hat{\mathbf{m}}$. In deriving Poisson's relation, we assumed that M and ρ are constant. However, we can consider a variable distribution of magnetization or density to be composed of arbitrarily small regions of uniform magnetization or density; equation 12.43 is appropriate for each of these small regions and, invoking the superposition principle, must be appropriate for variable distributions of density and magnetization.

Baranov [9] described an application of Poisson's relation in which the total-field magnetic anomaly is converted into the gravity anomaly that

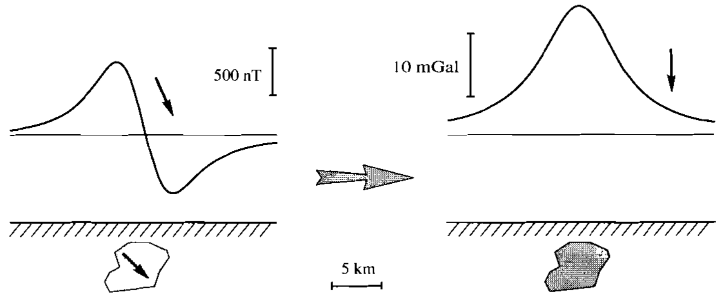

Fig. 12.11. A magnetic anomaly and its pseudogravity transform.

would be observed if the magnetization distribution were to be replaced with an identical density distribution (i.e., $\frac{M}{\rho}$ is a constant throughout the source). He called the resulting quantity a *pseudogravity anomaly*, and the transformation itself is generally referred to as a *pseudogravity transformation* (Figure 12.11). These are perhaps unfortunate names since mass is not involved in any way. As we shall see shortly, the transformation may more appropriately be considered to be a conversion from magnetic field to magnetic potential. Nevertheless, we will use the conventional term, pseudogravity, in the following discussion.

The pseudogravity transformation has several important applications. Some geologic units may be both highly magnetic and anomalously dense. A mafic pluton surrounded by sedimentary rocks, for example, may produce both a gravity and magnetic anomaly. A pseudogravity anomaly, calculated from the measured magnetic field, can be compared directly with measurements of the gravity field. Such comparisons might help to build an interpretation of the shape and size of the source, or at least permit an investigation of the ratio M/ρ and how it varies within the source (e.g., Kanasewich and Agarwal [145], Bott and Ingles [40], Cordell and Taylor [74], Chandler and Malek [55]).

A pseudogravity transformation might be a useful strategy in interpreting magnetic anomalies, not because we believe that a mass distribution actually corresponds to the magnetic distribution beneath the magnetic survey, but because gravity anomalies are in some ways more instructive and easier to interpret and quantify than magnetic anomalies. Gravity anomalies over tabular bodies have steepest horizontal gradients approximately over the edges of the bodies, and this property can be exploited in a magnetic interpretation by transforming the magnetic anomaly to a pseudogravity anomaly and searching the pseudogravity anomaly for maximum horizontal gradients. This application of the pseudogravity transform will be discussed in more detail in the next section.

The pseudogravity transform is more easily understood and more easily undertaken in the Fourier domain. Assuming that the ratio ρ/M is a constant at each point, the Fourier transform of equation 12.43 is given by

$$\mathcal{F}[g_{\mathrm{m}}] = -\frac{\gamma}{C_{\mathrm{m}}}\frac{\rho}{M}\mathcal{F}[V], \tag{12.44}$$

and combining with equation 12.25 provides

$$\mathcal{F}[g_{\mathrm{m}}] = \frac{\gamma}{C_{\mathrm{m}}|k|\Theta_{\mathrm{f}}}\frac{\rho}{M}\mathcal{F}[\Delta T].$$

This equation relates the total-field anomaly to one component of the gravity field, the component parallel to the magnetization. We are more interested in the vertical component of the gravity anomaly, however, and this can be found by dividing both sides by Θ_{m}. Hence, denoting the pseudogravity anomaly as ΔT_{psg}, we get

$$\mathcal{F}[\Delta T_{\mathrm{psg}}] = \mathcal{F}[\Delta T]\,\mathcal{F}[\psi_{\mathrm{psg}}], \tag{12.45}$$

where

$$\mathcal{F}[\psi_{\mathrm{psg}}] = \frac{\gamma}{C_{\mathrm{m}}|k|\Theta_{\mathrm{m}}\Theta_{\mathrm{f}}}\frac{\rho}{M}, \qquad |k| \neq 0, \tag{12.46}$$

and ρ/M is a constant. The function $\mathcal{F}[\psi_{\mathrm{psg}}]$ is a filter that transforms a total-field anomaly measured on a horizontal surface into the pseudogravity anomaly. As we have seen in previous sections of this chapter, the transformation amounts to a three-step procedure: Fourier transform the total-field anomaly, multiply by $\mathcal{F}[\psi_{\mathrm{psg}}]$, and inverse Fourier transform the product. Subroutine B.27 in Appendix B is an implementation of this three-step transformation.

Notice the similarities between $\mathcal{F}[\psi_{\mathrm{psg}}]$ and the reduction-to-pole filter $\mathcal{F}[\psi_{\mathrm{r}}]$. In particular, the two filters are related by

$$\mathcal{F}[\psi_{\mathrm{psg}}] = \frac{A}{|k|}\mathcal{F}[\psi_{\mathrm{r}}],$$

where A is a constant. Thus the two filters have certain spectral properties in common. Indeed, the phase spectrum of $\mathcal{F}[\psi_{\mathrm{psg}}]$ is identical to that of $\mathcal{F}[\psi_{\mathrm{r}}]$ (Figure 12.9), and we can expect the pseudogravity transformation to have limitations when the magnetization and ambient field have shallow inclinations, as might be expected at low latitudes. The amplitude spectrum of $\mathcal{F}[\psi_{\mathrm{psg}}]$ is proportional to the amplitude spectrum of $\mathcal{F}[\psi_r]$ (Figure 12.9) weighted by $1/|k|$. Hence, the radial amplitude spectrum is proportional to $1/|k|$; that is, the pseudogravity transformation amplifies low wavenumbers (long wavelengths) and attenuates high wavenumbers (short wavelengths). The low-wavenumber amplification is cause for some concern; any long-wavelength noise contained in the measured total-field data will be amplified along with authentic anomalies.

Also note the relationship between pseudogravity and magnetic potential,

$$\mathcal{F}[g_{\text{psg}}] = \frac{B}{\Theta_{\text{m}}} \mathcal{F}[V],$$

where B is a constant. In particular, the pseudogravity anomaly of a magnetic source is proportional to the magnetic potential of the same source with vertical magnetization.

Figure 12.12 shows the pseudogravity transform of the total-field anomaly from central Nevada. Note its similarities with the upward-continued field (Figure 12.4). In particular, long-wavelength features of the original map have been amplified at the expense of short-wavelength anomalies.

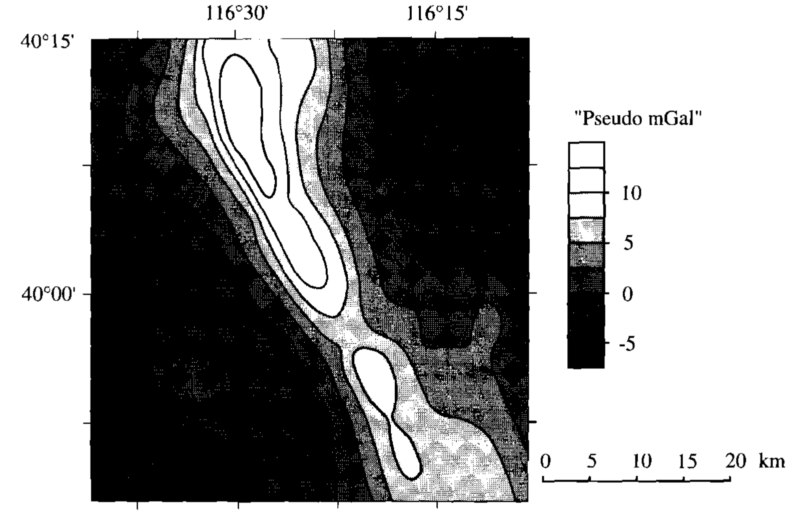

Fig. 12.12. The pseudogravity anomaly transformed from the total-field anomaly of Figure 12.1.

12.4.1 Pseudomagnetic Calculation

The inverse of equation 12.45 can be used to transform a measured gravity anomaly into the magnetic anomaly that would be observed if the density distribution were replaced by a magnetic distribution in one-to-one proportion. Rearranging equation 12.45 yields

$$\mathcal{F}[g_{\mathrm{psm}}] = \mathcal{F}[g]\mathcal{F}[\psi_{\mathrm{psm}}], \tag{12.47}$$

where g_{psm} denotes the transformed anomaly and

$$\mathcal{F}[\psi_{\mathrm{psm}}] = \frac{C_{\mathrm{m}}|k|\Theta_{\mathrm{m}}\Theta_{\mathrm{f}}}{\gamma}\frac{M}{\rho}\,. \tag{12.48}$$

This operation, called a *pseudomagnetic transformation*, does not suffer from the low-latitude limitations of its pseudogravity counterpart, but it clearly can suffer from instabilities. In particular, short-wavelength components of g_z are amplified; the shorter the wavelength, the greater will be the amplification. Any noise present at these wavelengths will be similarly amplified, and this can lead to high-amplitude, short-wavelength artifacts in the transformed result.

12.5 Horizontal Gradients and Boundary Analysis

The steepest horizontal gradient of a gravity anomaly $g_z(x, y)$ (or of a pseudogravity anomaly) caused by a tabular body tends to overlie the edges of the body. Indeed, the steepest gradient will be located directly over the edge of the body if the edge is vertical and far removed from all other edges or sources.

Exercise 12.6 Consider a uniform, horizontal, semi-infinite slab with vertical face (i.e., the slab occupies the region $0 \le x < \infty$, $-\infty < y < \infty$, $z_1 < z < z_2$), like that shown in Figures 9.15(a) and 9.15(b). Show from equation 9.2.2 that a gravity profile over the edge of the slab measured in the x direction will have its maximum gradient over the edge of the slab.

We can exploit this characteristic of gravity anomalies in order to locate abrupt lateral changes in density directly from gravity measurements (Cordell [66]). Moreover, the same technique could be applied to magnetic measurements by first transforming them to pseudogravity anomalies, in which case the steepest horizontal gradients would reflect abrupt lateral changes in magnetization (Cordell and Grauch [68, 70]).

The *magnitude of the horizontal gradient* of the gravity or pseudogravity anomaly, loosely referred to here as the *horizontal gradient*, is