Designing a Total Field Magnetic Survey over Various Sources

- Knowing the expected source types is important

- Given expectations for sources, we can determine sample spacing and line spacing

- Standard sampling theory requires at least two samples per the shortest wavelength of signal in any gridded or profiled data set

- Under-sampling the power spectrum aliases long wavelength signal into the results

- Under-sampled data sets are not appropriate for modeling or inversion

- High sample density along lines does not add significant cost

- Using the power spectrum of expected sources we determine the ratio of elevation above sources and spacing between lines of acquired data

Sampling Theory for Field Work Involving Spatial Distributions

The Nyquist wavelength (λ_N) is the shortest spatial wavelength that can be accurately recovered in field work by sequential observations with spacing Δx:

$$\lambda_N = 2\,\Delta x$$

- If wavelengths shorter than λ_N exist in the field area:
  - they do not appear in the measured results
  - their power will be aliased into wavelengths longer than λ_N
  - the resulting anomalies contain more power in longer wavelengths than actually exists
  - the results are inappropriate for modeling or inversion

The magnetic (gravity, etc.) field being sampled must contain no significant components shorter than λ_N or they are aliased into long wavelength components

Choosing the appropriate Nyquist wavelength (λ_N) for fieldwork

In designing your field work, you need to choose **Δx.** This yields:

- the Nyquist wavelength

  $$\lambda_N = 2\,\Delta x$$

- and the Nyquist wavenumber

  $$k_N = 2\pi / \lambda_N$$

  $$k_N = \pi / \Delta x.$$

The Nyquist wavenumber is the largest wavenumber in your Fourier transformed results as it comes from the shortest wavelength

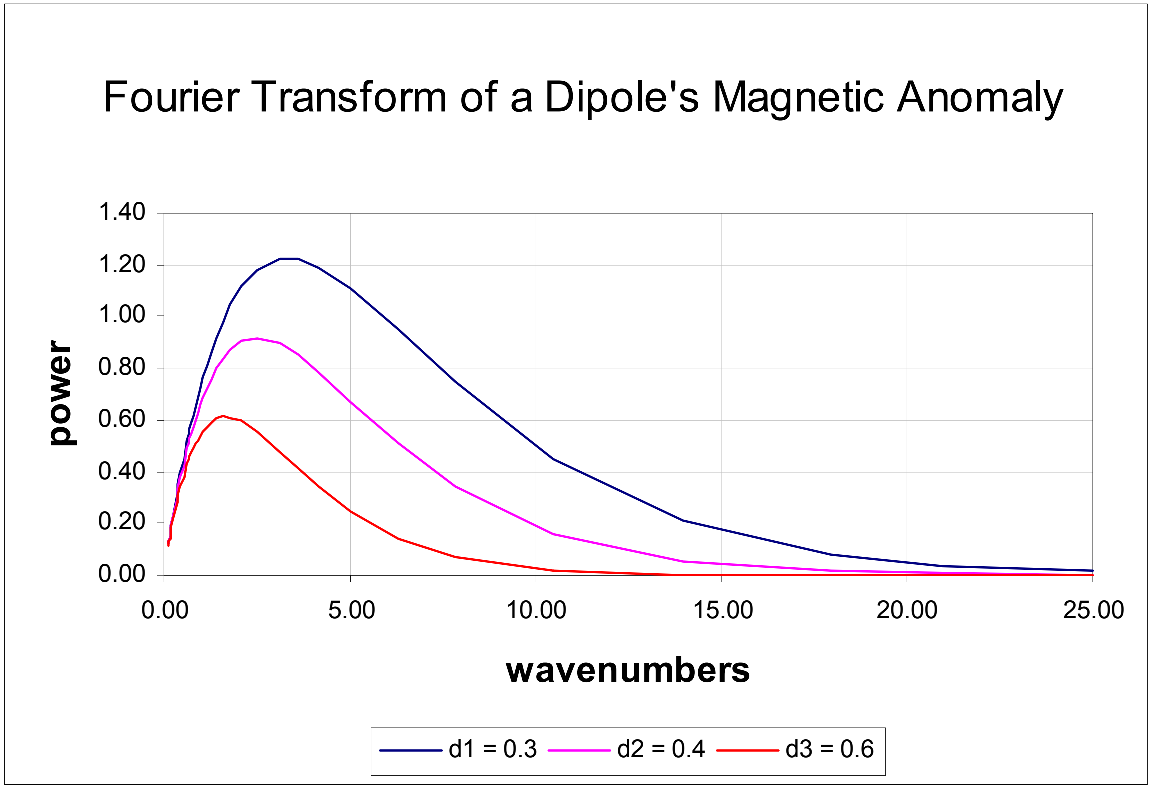
Next – apply this to sampling the field over a buried dipole

The Fourier transform of the magnetic anomaly from a dipole is:

$$F(T) = 2\pi M C_m \theta_m \theta_f \,|k|\, e^{|k|(z_o - z')} \quad \text{(from Blakely, 1995)}$$

Z_o = elevation of planar observation surface
Z' = depth of dipole below the observation surface
M = magnetization
Cm contains units of magnetization
θ_m = magnetization parameters
θ_f = field parameters
$z' > z_0$

All the depth information {f(z)} is contained in the exponential term.



The Fourier transform of the magnetic anomaly from a dipole is:

$$F(T) = 2\pi M C_m \theta_m \theta_f \,|\,k\,|\, e^{|k|(z_o - z')}$$

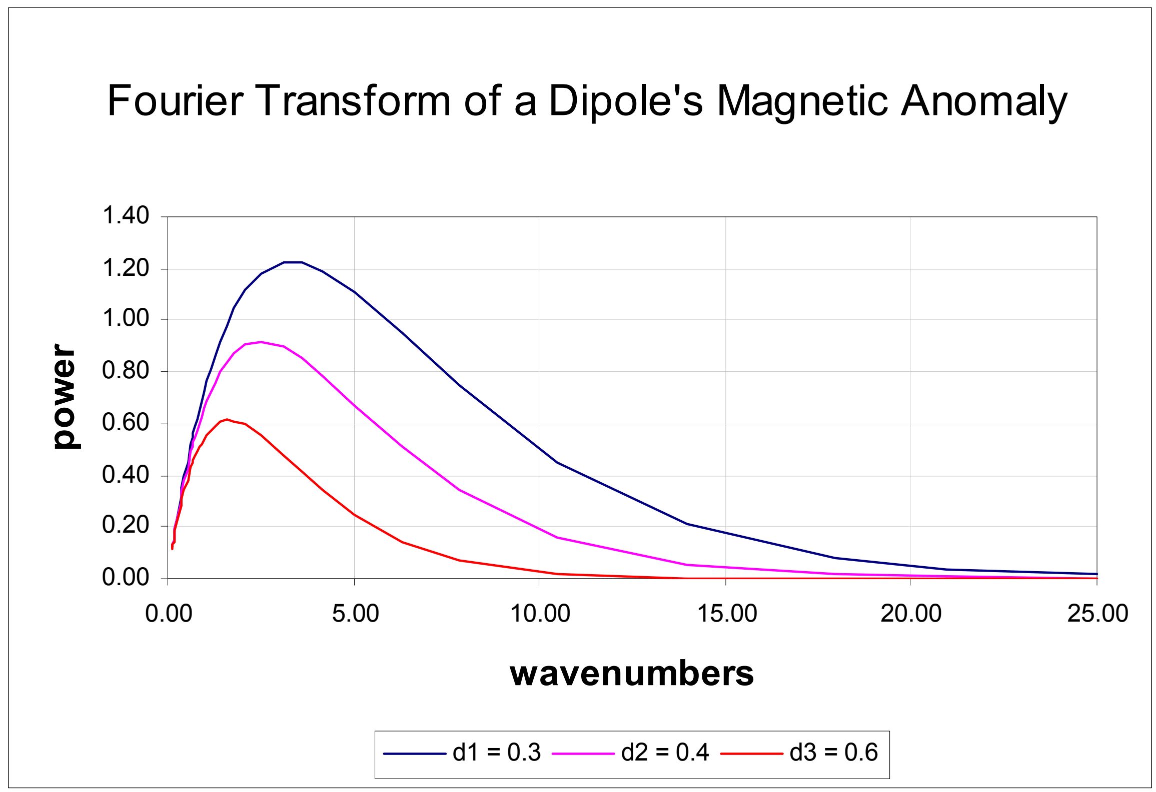
The maximum energy of the total field anomaly when plotted against wavenumbers (2π/wavelength) depends on the depth of the dipole.

Differentiating with respect to k shows that $Z \approx 1/k_{max}$, where k_{max} is the wavenumber of maximum power.

Here, the maximum for depth d1 occurs near wavenumber ~ 3.1

thus depth ~ 0.3

The spectrum is clearly sensitive to the source parameters



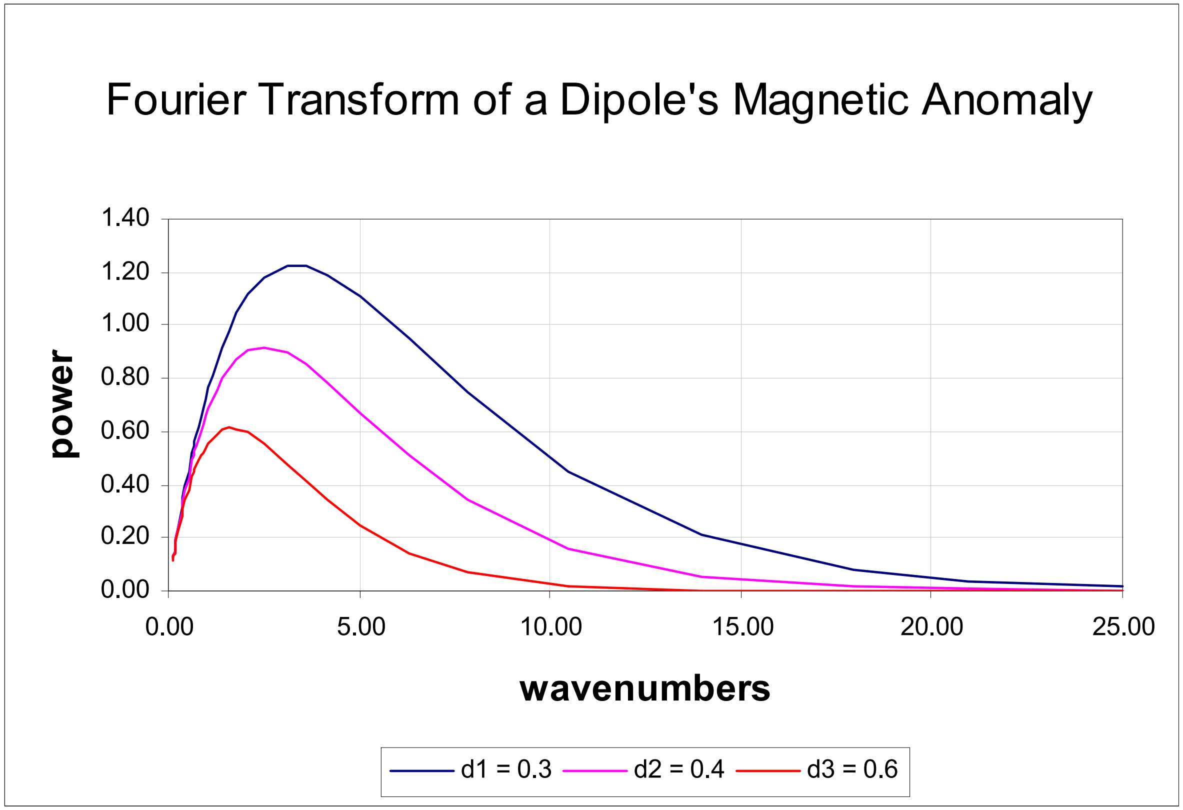
The Fourier transform of the magnetic anomaly from a dipole is:

$$F(T) = 2\pi M C_m \theta_m \theta_f \, |k| \, e^{|k|(z_o - z')}$$

To simplify for analysis:

- assume induced magnetization at the magnetic pole & group all the constants
- let -z = (zo – z'), the height of the sensor above the dipole (this includes survey or flight height):

$$F(T_p) = C_1 \, |k| \, e^{|k|(-z)}$$



Consider a buried dipole somewhere in your field area

from before:

$$F(T_p) = C_1 \, | k | \, e^{-|k|z}$$

and its power spectrum is:

$$F(T_p) = C \, | k^2 | \, e^{-2|k|z}$$

If it is under-sampled, the fraction of the shorter than Nyquist wavelengths aliased into your measured results will be:

$$F_a = \frac{\int_{k_N}^{\infty} C \, |k| \, e^{-2|k|z}}{\int_{0}^{\infty} C \, | k^2 | \, e^{-2|k|z}}$$ where the indefinite integral is:

$$\int C \, | k^2 | \, e^{-2|k|z} = \frac{-Ce^{-2|k|z}(2k^2z^2 + 2kz + 1)}{4z^3}$$

The Aliased Signal from an Under-Sampled, Buried Dipole

Then:

$$F_a = \frac{\int_{k_N}^{\infty} C|k|e^{-2|k|z}}{\int_{0}^{\infty} C|k^2|e^{-2|k|z}} = \frac{\dfrac{Ce^{-2k_n z}(2k_n z(k_n z+1)+1)}{4z^3}}{\dfrac{C}{4z^3}}, \text{ and}$$

the fraction of the shorter than Nyquist wavelengths aliased into your measured results will be:

$$\boxed{F_a = e^{-2k_n z}(2k_n z(k_n z+1)+1)}$$

The Aliased Signal from an Under-Sampled, Buried Dipole

We have:

$$F_a = e^{-2k_n z}(2k_n z(k_n z + 1) + 1)$$

and know the Nyquist wavenumber is:

$$k_N = \pi / \Delta x$$

So, substitute and

$$F_a = e^{-\frac{2\pi z}{\Delta x}}(\frac{2\pi z}{\Delta x}(\frac{\pi z}{\Delta x} + 1) + 1)$$

The Aliased Signal from an Under-Sampled, Buried Dipole

$$F_a = e^{-\frac{2\pi z}{\Delta x}} \left(\frac{2\pi z}{\Delta x} \left(\frac{\pi z}{\Delta x} + 1 \right) + 1 \right)$$

is now expressed in the parameters of field work:

- z is depth to source below the sensor
- Δx is the line or sample spacing
- F_a is the fraction of power aliased from short wavelengths to longer wavelengths when the signal is under-sampled with sample spacing starting at k_N.

Buried Dipole - How does the fraction of aliased power vary with source depth (z) and sample (or line) spacing (Δx)?

Archaeological surveys with pertinent depth ranges below the sensor of:

z: 0.4 to 2 meters

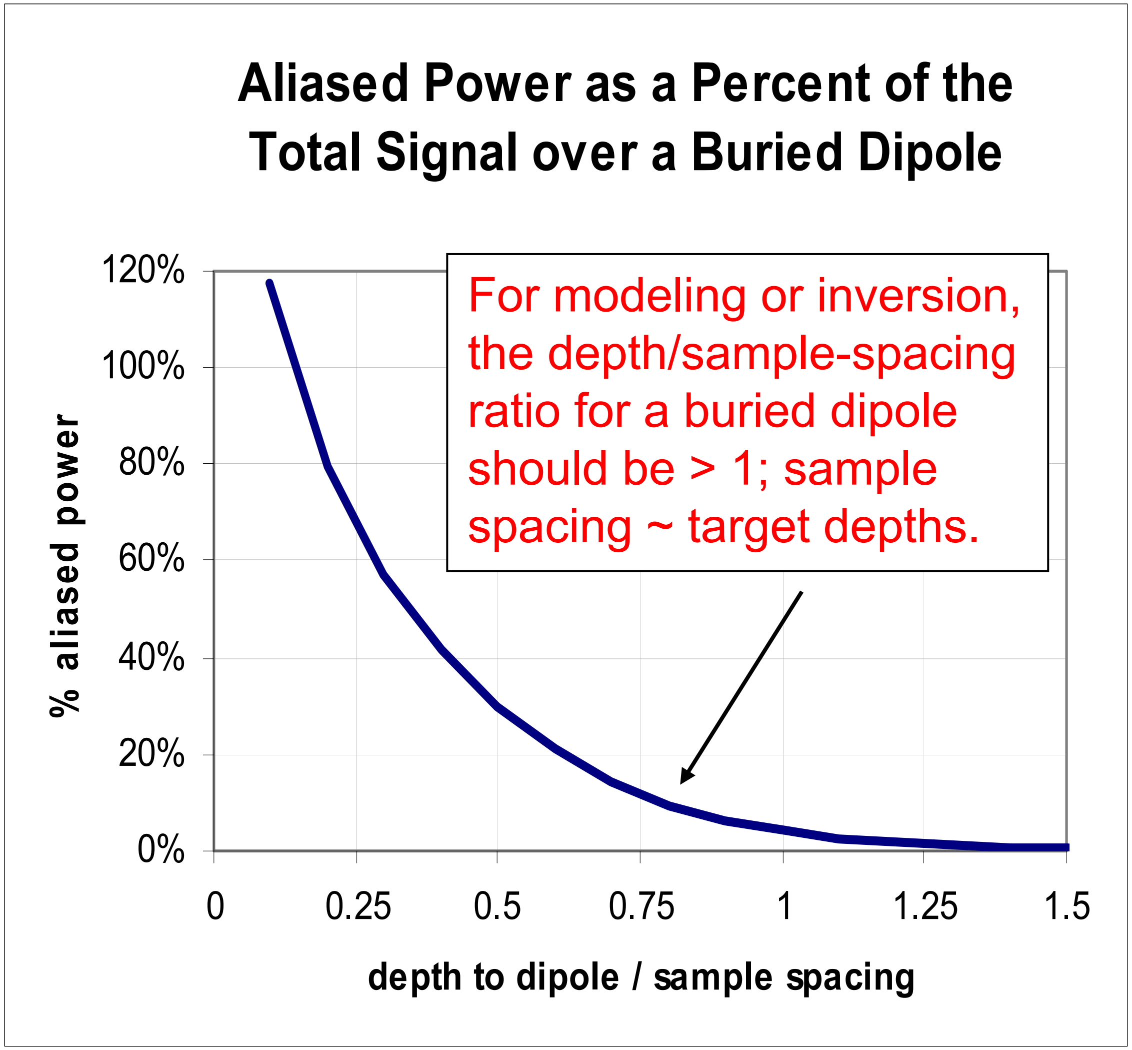
should use line/sample spacing:

Δx <= 0.4 meters

Minerals surveys with targets from 100 to 300 meters below the sensor should use line/sample spacing:

Δx <= 100 meters.

In either case, larger spacing increases the aliasing and decreases the reliability of modeling and inverse results.



Fourier Transform of a Randomly Magnetized Layer:

$$F(T_L) = F(M_L) * \{2\pi C_m \theta_m \theta_f \, e^{|k|z_o} (e^{-|k|z_1} - e^{-|k|z_2})\} \text{ (Blakely, 1995)}$$

Z_0 = elevation of planar observation surface
Z_1 = depth to top of layer
Z_2 = depth to bottom of layer
M_L = magnetization of the layer
Cm contains units of magnetization
θ_m = magnetization parameters
θ_f = magnetic field parameters

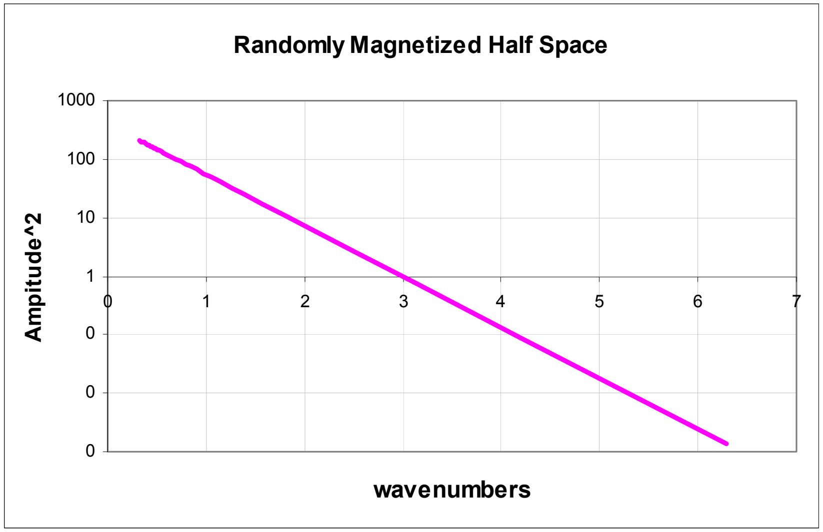
To simplify:

- assume $z_0 = 0$
- assume induced magnetization
- group all constants
- let z_2 go to infinity to get a randomly magnetized half space

$$F(T_h) = F(M_h) * \{C_1 \, e^{-|k|z_1}\}$$

How do we sample the field over a half space at depth z_1?

$$F(T_h) = F(M_h) * \{C_1 e^{-|k|z_1}\}$$



Its power spectrum is:

$$F(T_h) = F(M) * \{C e^{-2|k|z_1}\}$$

If it is under-sampled, the fraction of the shorter than Nyquist wavelengths aliased into your measured results will be:

$$F_a = \frac{\int_{k_N}^{\infty} F(M) * \{C e^{-2|k|z_1}\}}{\int_{0}^{\infty} F(M) * \{C e^{-2|k|z_1}\}}$$

Aliased Signal From a Poorly Sampled Magnetic Half Space:

$$F_a = \frac{\int_{k_N}^{\infty} F(M) * \{C e^{-2|k|z_1}\}}{\int_{0}^{\infty} F(M) * \{C e^{-2|k|z_1}\}}$$

The indefinite integral is:

$$\int F(M) * \{C e^{-2|k|z_1}\} = F(M) \frac{-Ce^{-2|k|z_1}}{2k}$$

Aliased Signal From a Poorly Sampled Magnetic Half Space:

$$Fa = \frac{\int_{k_N}^{\infty} F(M)\,\frac{-Ce^{-2|k|z_1}}{2k}}{\int_{0}^{\infty} F(M)\,\frac{-Ce^{-2|k|z_1}}{2k}} = \frac{\int_{k_N}^{\infty} \frac{-Ce^{-2|k|z_1}}{2k}}{\int_{0}^{\infty} \frac{-Ce^{-2|k|z_1}}{2k}}, \text{ then}$$

$$F_a = \frac{\frac{Ce^{-2k_n z_1}}{2z_1}}{\frac{C}{2z_1}}$$

$$\boxed{F_a = e^{-2k_n z_1}}$$

Aliased Signal From a Poorly Sampled Magnetic Half Space:

$$F_a = e^{-2k_n z_1}$$

This is the fraction of shorter than Nyquist wavelengths aliased into your field observations. Now put them into experimental parameters.

The Nyquist wavenumber is:

$$k_N = \pi / \Delta x$$

So, substitute and

$$F_a = e^{-\frac{2\pi z_1}{\Delta x}}$$

Aliased Signal From a Poorly Sampled Magnetic Half Space:

$$F_a = e^{-\frac{2\pi z_1}{\Delta x}}$$

Is now expressed in the parameters of field work:

- z_1 is depth from the sensor to the top of the layer
- Δx is the line or sample spacing
- F_a is the fraction of power aliased from short wavelengths to longer wavelengths when the signal is under-sampled with sample spacing starting at $k_{N.}$

Magnetized Half Space - How does the fraction of aliased power vary with layer depth (z) and sample (or line) spacing (Δx)?

Archaeological surveys with pertinent depth ranges below the sensor of:

z: 0.4 to 2 meters

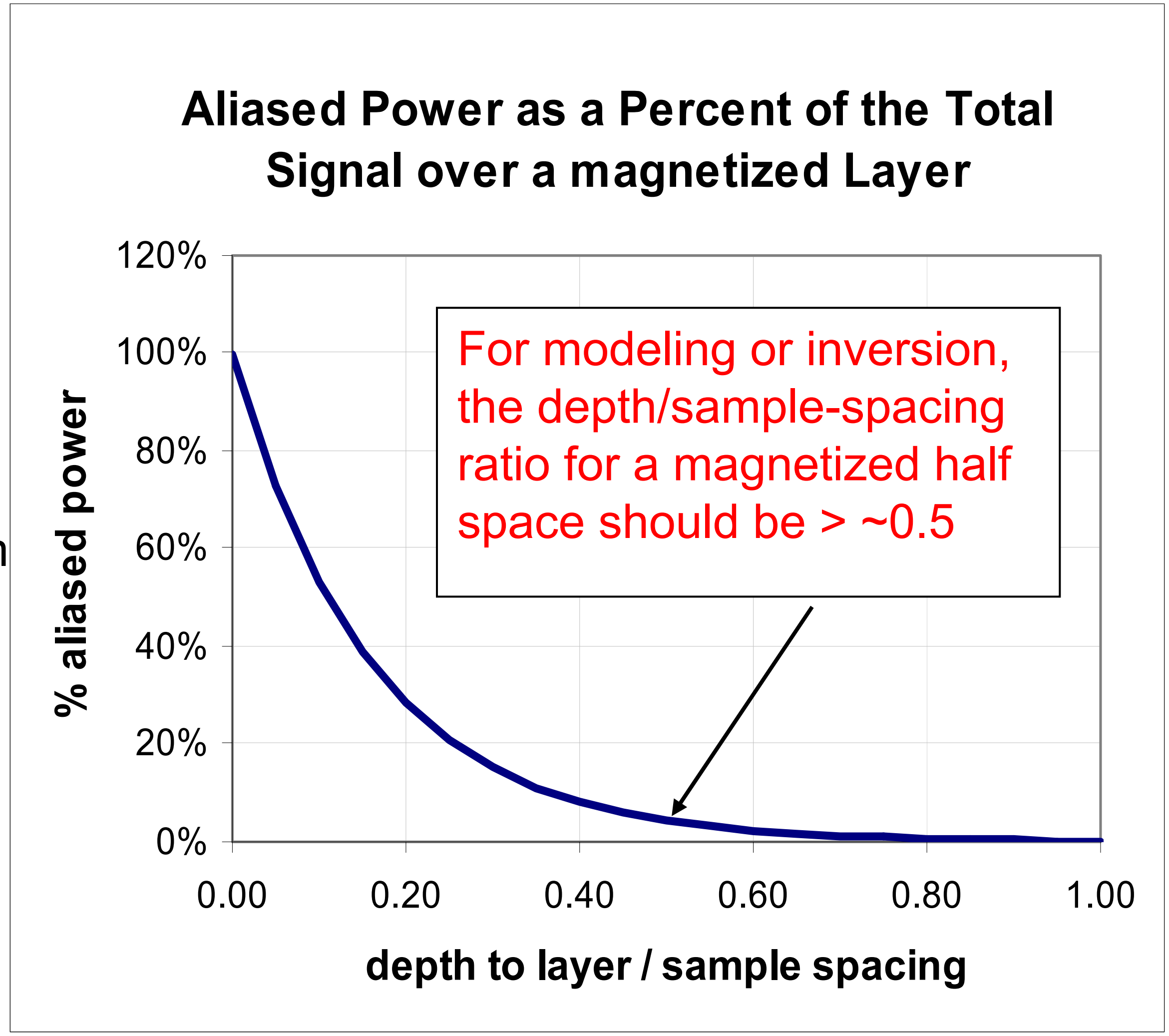
should use line/sample spacing:

Δx <= 0.8 meters

Minerals surveys with targets from 100 to 300 meters below the sensor should use line/sample spacing:

Δx <= 200 meters.

In either case, larger spacing increases the aliasing and decreases the reliability of modeling and inverse results.



Choosing Sample or Line Spacing for Magnetic Fieldwork

• We can assess the fraction of aliased power due to under sampling short wavelengths by analyzing appropriate power spectra

• Individual sources, as represented by a buried dipole, require line or sample spacing equal to or less than the depth of the source below the sensor

• A randomly magnetized half space has less stringent requirements. Here, sample or line spacing can be about twice the layer depth

• In either case, under sampling the field by choosing too large of sample or line spacing adds amplitude to longer wavelength anomalies

•Under-sampled results are not suitable for accurate modeling or inversion.