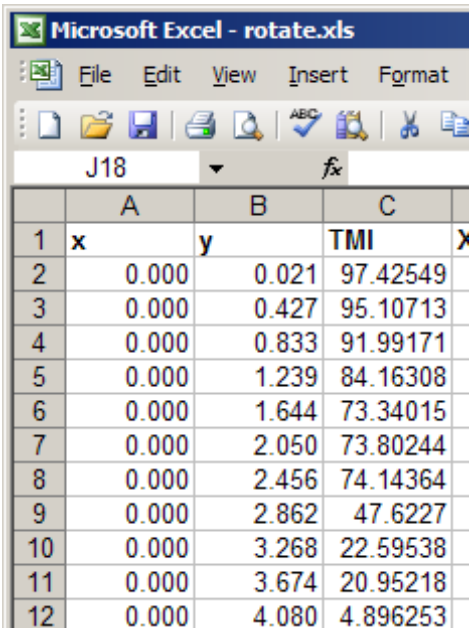
Rotating Field Coordinates into GPS or Total Station Coordinates

Assumptions:

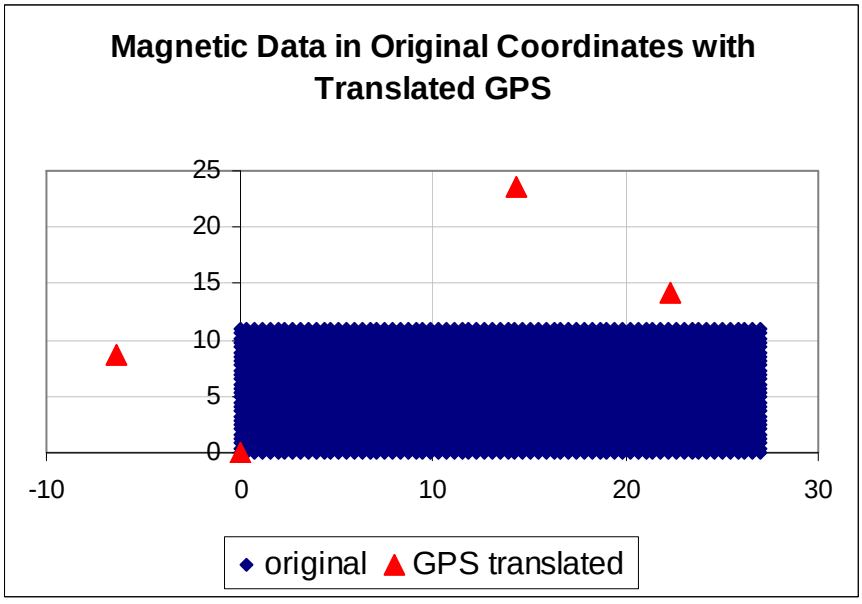
- Magnetic grids are collected in a rectangular grid starting at X=0, Y=0.
- The corners are known in another coordinate system (e.g. GPS or total station); axes need not be aligned.
- The starting spreadsheet has X, Y, and TMI as in the figure to the right.



	A	B	C
1	x	y	TMI
2	0.000	0.021	97.42549
3	0.000	0.427	95.10713
4	0.000	0.833	91.99171
5	0.000	1.239	84.16308
6	0.000	1.644	73.34015
7	0.000	2.050	73.80244
8	0.000	2.456	74.14364
9	0.000	2.862	47.6227
10	0.000	3.268	22.59538
11	0.000	3.674	20.95218
12	0.000	4.080	4.896253

Steps:

- Make room for 4 additional columns
  1. X rotated
  2. Y rotated
  3. X translated
  4. Y translated
- Add three tables to the spreadsheet:
  1. GPS corners
  2. Grid Corners
  3. Rotated grid corners (to be filled in later)
- Translate the GPS coordinates to the origin:
  1. X_translated = X – southwest corner X value
  2. Y_translated = Y – southwest corner Y value
- Make a map (graph) of the original data points and the translated GPS points

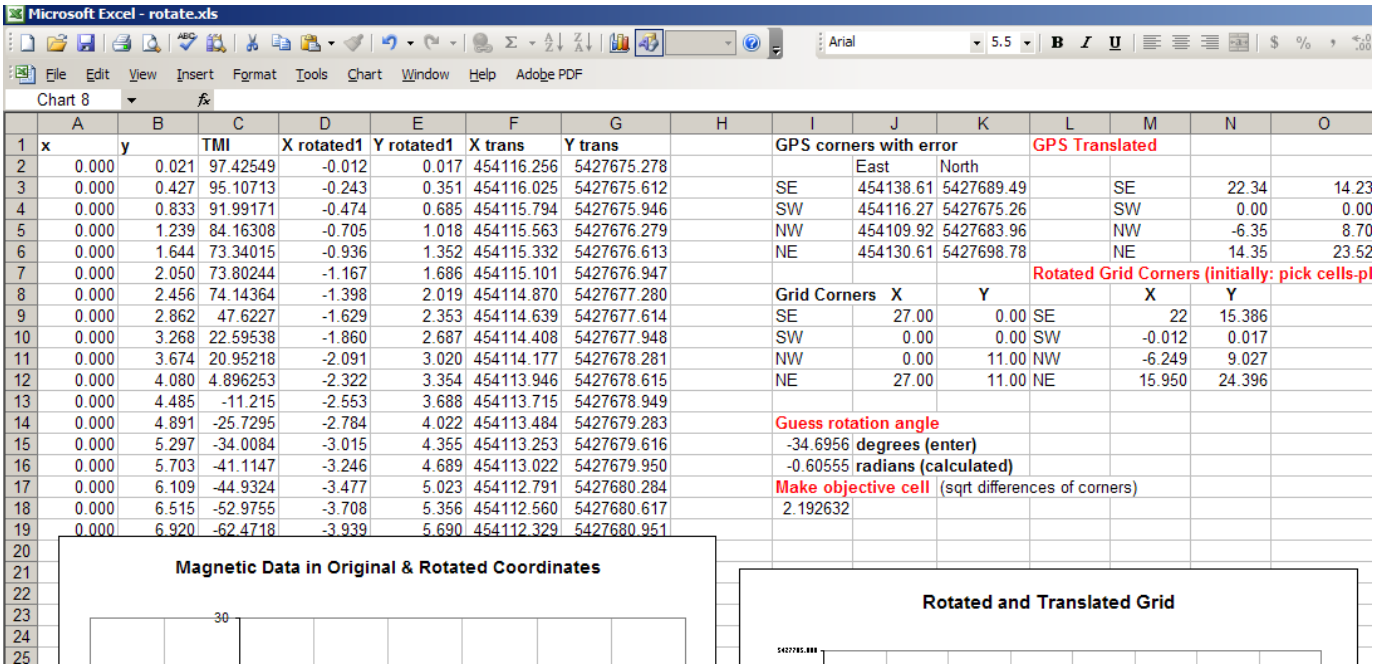


- Look at the figure and guess the needed amount of rotation. In this example it is about 30^{o}.

- Using the X_rotated and Y_rotated columns, rotate the original (x, y) values by the guessed amount of rotation. The rotation equations are:

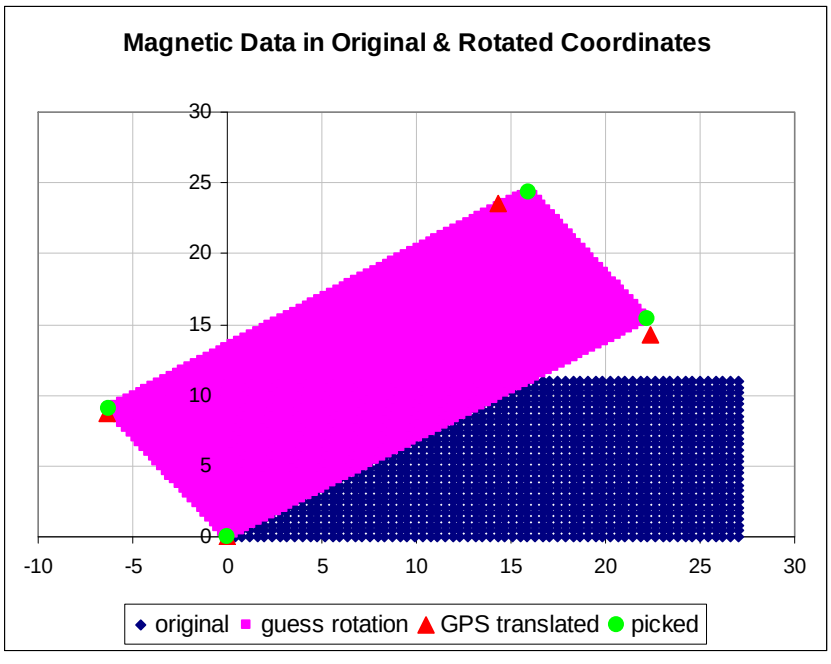
$$X' = x*COS(Rot\_angle) + y*SIN(Rot\_angle)$$
$$Y' = -x*SIN(Rot\_angle) + y*COS(Rot\_angle)$$

remember that Excel requires angles in radians when using trigonometric functions.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	x	y	TMI	X rotated1	Y rotated1	X trans	Y trans		GPS corners with error			GPS Translated			
2	0.000	0.021	97.42549	-0.012	0.017	454116.256	5427675.278			East	North				
3	0.000	0.427	95.10713	-0.243	0.351	454116.025	5427675.612		SE	454138.61	5427689.49		SE	22.34	14.23
4	0.000	0.833	91.99171	-0.474	0.685	454115.794	5427675.946		SW	454116.27	5427675.26		SW	0.00	0.00
5	0.000	1.239	84.16308	-0.705	1.018	454115.563	5427676.279		NW	454109.92	5427683.96		NW	-6.35	8.70
6	0.000	1.644	73.34015	-0.936	1.352	454115.332	5427676.613		NE	454130.61	5427698.78		NE	14.35	23.52
7	0.000	2.050	73.80244	-1.167	1.686	454115.101	5427676.947					Rotated Grid Corners (initially: pick cells-pl			
8	0.000	2.456	74.14364	-1.398	2.019	454114.870	5427677.280		Grid Corners	X	Y		X	Y	
9	0.000	2.862	47.6227	-1.629	2.353	454114.639	5427677.614		SE	27.00	0.00	SE	22	15.386	
10	0.000	3.268	22.59538	-1.860	2.687	454114.408	5427677.948		SW	0.00	0.00	SW	-0.012	0.017	
11	0.000	3.674	20.95218	-2.091	3.020	454114.177	5427678.281		NW	0.00	11.00	NW	-6.249	9.027	
12	0.000	4.080	4.896253	-2.322	3.354	454113.946	5427678.615		NE	27.00	11.00	NE	15.950	24.396	
13	0.000	4.485	-11.215	-2.553	3.688	454113.715	5427678.949								
14	0.000	4.891	-25.7295	-2.784	4.022	454113.484	5427679.283		Guess rotation angle						
15	0.000	5.297	-34.0084	-3.015	4.355	454113.253	5427679.616		-34.6956	degrees (enter)					
16	0.000	5.703	-41.1147	-3.246	4.689	454113.022	5427679.950		-0.60555	radians (calculated)					
17	0.000	6.109	-44.9324	-3.477	5.023	454112.791	5427680.284		Make objective cell	(sqrt differences of corners)					
18	0.000	6.515	-52.9755	-3.708	5.356	454112.560	5427680.617		2.192632						
19	0.000	6.920	-62.4718	-3.939	5.690	454112.329	5427680.951								

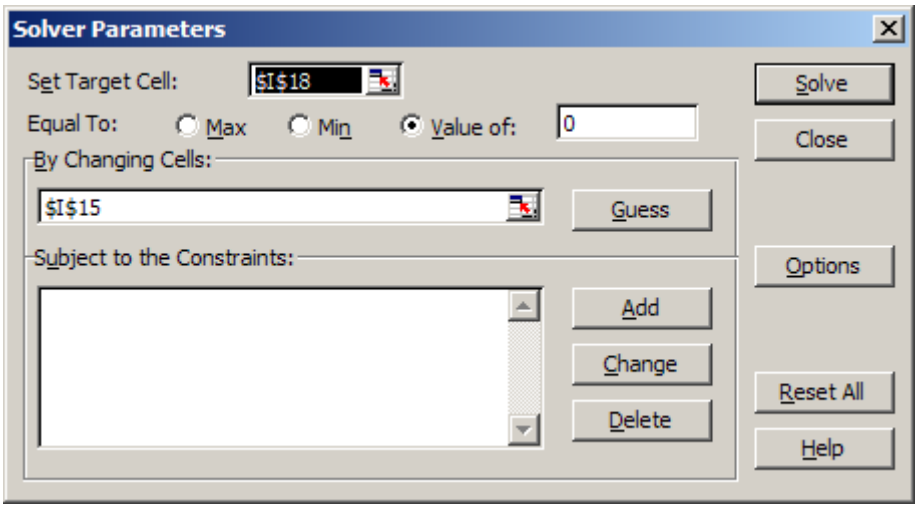
- Plot the rotated values and adjust the rotation angle visually until the grid fits well.



- Fill in the 'Rotated Grid Corners' table (‘picked’ in figure) by pointing to appropriate cells in the rotated columns.

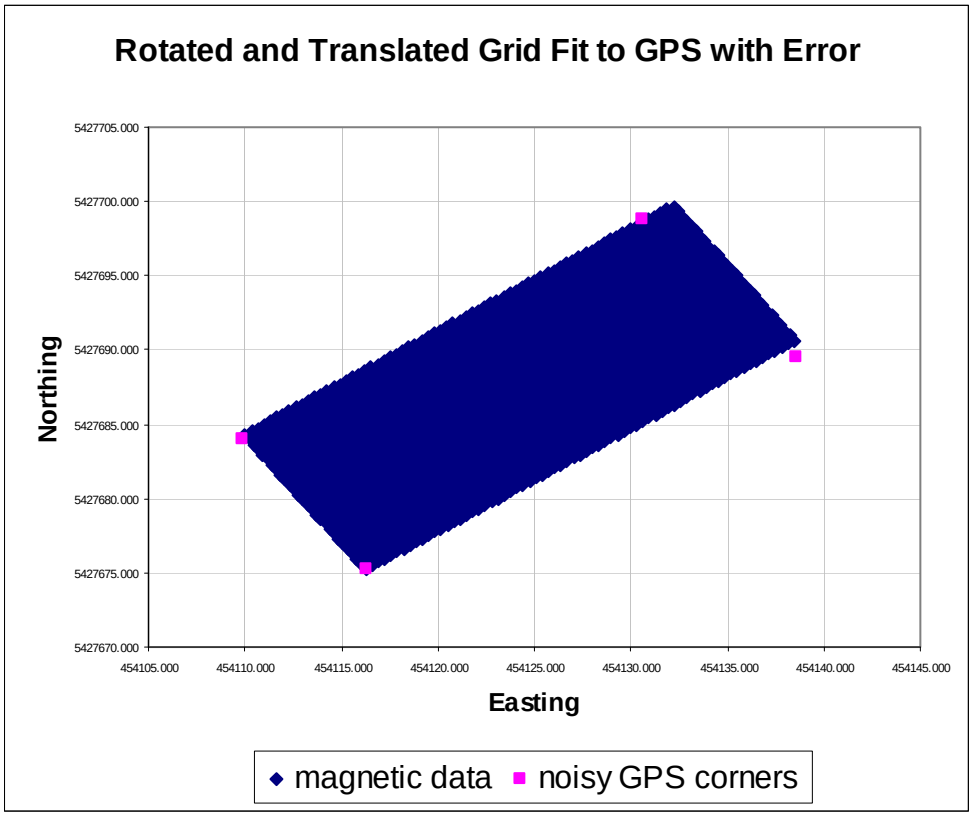
- To use Excel’s SOLVER for nonlinear least squares improvement of your guess make a target cell with an objective function. I use the square root of the sum of the differences of each of the coordinate pairs:

  =SQRT((M9-N3)^2+(M10-N4^2)+(M11-N5)^2+(M12-N6)^2+(N9-O3)^2+(N10-O4)^2+(N11-O5)^2+(N12-O6)^2)



- Following SOLVER, undo the translation step above by translating the rotated grid back to the GPS origin:

1. X_translated = X + southwest corner GPS_X value
2. Y_translated = Y + southwest corner GPS_Y value



Microsoft Excel - rotate.xls

File Edit View Insert Format Tools Chart Window Help Adobe PDF

Chart Area

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	x	y	TMI	X rotated1	Y rotated1	X trans	Y trans		GPS corners with error			GPS Translated				
2	0.000	0.021	97.42549	-0.012	0.017	454116.256	5427675.278			East	North					
3	0.000	0.427	95.10713	-0.243	0.351	454116.025	5427675.612		SE	454138.61	5427689.49		SE	22.34	14.23	
4	0.000	0.833	91.99171	-0.474	0.685	454115.794	5427675.946		SW	454116.27	5427675.26		SW	0.00	0.00	
5	0.000	1.239	84.16308	-0.705	1.018	454115.563	5427676.279		NW	454109.92	5427683.96		NW	-6.35	8.70	
6	0.000	1.644	73.34015	-0.936	1.352	454115.332	5427676.613		NE	454130.61	5427698.78		NE	14.35	23.52	
7	0.000	2.050	73.80244	-1.167	1.686	454115.101	5427676.947					Rotated Grid Corners (initially: pick cells-plot to check)				
8	0.000	2.456	74.14364	-1.398	2.019	454114.870	5427677.280		Grid Corners	X	Y		X	Y		
9	0.000	2.862	47.6227	-1.629	2.353	454114.639	5427677.614		SE	27.00	0.00	SE	22	15.386		
10	0.000	3.268	22.59538	-1.860	2.687	454114.408	5427677.948		SW	0.00	0.00	SW	-0.012	0.017		
11	0.000	3.674	20.95218	-2.091	3.020	454114.177	5427678.281		NW	0.00	11.00	NW	-6.249	9.027		
12	0.000	4.080	4.896253	-2.322	3.354	454113.946	5427678.615		NE	27.00	11.00	NE	15.950	24.396		
13	0.000	4.485	-11.215	-2.553	3.688	454113.715	5427678.949									
14	0.000	4.891	-25.7295	-2.784	4.022	454113.484	5427679.283		Guess rotation angle							
15	0.000	5.297	-34.0084	-3.015	4.355	454113.253	5427679.616		-34.6956	degrees (enter)						
16	0.000	5.703	-41.1147	-3.246	4.689	454113.022	5427679.950		-0.60555	radians (calculated)						
17	0.000	6.109	-44.9324	-3.477	5.023	454112.791	5427680.284		Make objective cell	(sqrt differences of corners)						
18	0.000	6.515	-52.9755	-3.708	5.356	454112.560	5427680.617		2.192632							
19	0.000	6.920	-62.4718	-3.939	5.690	454112.329	5427680.951									
20	0.000	7.326	-74.8473	-4.170	6.024	454112.098	5427681.285									
21	0.000	7.732	-88.8532	-4.401	6.357	454111.867	5427681.618									
22	0.000	8.138	-98.386	-4.632	6.691	454111.636	5427681.952									
23	0.000	8.544	-107.07	-4.863	7.025	454111.405	5427682.286									
24	0.000	8.950	-111.546	-5.094	7.358	454111.174	5427682.619									
25	0.000	9.356	-115.541	-5.325	7.692	454110.943	5427682.953									
26	0.000	9.761	-113.074	-5.556	8.026	454110.712	5427683.287									
27	0.000	10.167	-117.094	-5.787	8.359	454110.481	5427683.620									
28	0.000	10.573	-121.107	-6.018	8.693	454110.250	5427683.954									
29	0.000	10.979	-125.397	-6.249												
30	0.397	0.021	102.5852	0.315												
31	0.397	0.427	95.04312	0.083												
32	0.397	0.833	91.07493	-0.148												
33	0.397	1.239	83.93953	-0.379												
34	0.397	1.644	72.97114	-0.610												
35	0.397	2.050	67.95383	-0.841												
36	0.397	2.456	63.48991	-1.072												
37	0.397	2.862	45.35386	-1.303												
38	0.397	3.268	26.26563	-1.534												
39	0.397	3.674	17.85457	-1.765												
40	0.397	4.080	4.763398	-1.996												
41	0.397	4.485	-9.4998	-2.227												
42	0.397	4.891	-22.4906	-2.458