

## Error Analysis, Uncertainties, Propagation of Uncertainties

Error analysis and the propagation of uncertainty in calculations is an important topic in any experiment be it field and/or lab work. The topic is formally addressed and very well presented in: *An Introduction to Error Analysis: The study of Uncertainties in Physical Measurements*, J. R. Taylor, 2nd edition, University Science Books, 1997, 328 - an excellent addition to any scientist's bookshelf. Taylor's text is the standard, and I've borrowed heavily from it for this short outline; you should refer to Taylor for a full and entertaining discussion of the subject.

### Best Estimate

If we measure a quantity several times, usually the best estimate of that quantity will be the average of the measurements. Along with your best estimate of a quantity you should report the uncertainty associated with that estimate and the method you used to determine it.

Suppose six students make six successive readings from the gravimeter: 1532.4, 1532.4, 1532.3, 1531.3, 1532.5, 1532.5. The best estimate (average  $\bar{x}$ ) of the meter reading is then: 1532.4, found, in general, with the formula:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

where  $N$  is the number of observations and  $x_i$  represents the observations.

### Uncertainty

If we have a number of observations  $x_i$ , then each will have some deviation from the mean. The standard deviation, sometimes called the root mean square error, is:

$$SD = s_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

The standard deviation of the meter readings above is 0.089 and should be reported as 0.1. You should almost always round experimental uncertainties to one significant figure unless that figure is a one – then use two significant figures. Another standard to follow is that the last significant figure in your best estimate should usually be of the same order of magnitude (same decimal position) as the uncertainty. Thus, if you had a mean elevation determination by GPS of 982.34 meters with a standard deviation of 5.0 meters, you would report  $982 \pm 5$  meters not  $982.34 \pm 5$  meters as the 0.34 meters is meaningless.

The gravimeter readings would now be reported like this: "... the gravimeter reading was  $1532.4 \pm 0.1$  scale divisions ..."

### Fractional Uncertainty

Fractional uncertainty, reporting the uncertainty by percent, is also fairly common. Fractional uncertainty is calculated by:

$$\text{Fractional uncertainty} = \frac{\text{uncertainty (} \mathbf{s}_x \text{)}}{\bar{x}}$$

### Propagation of Uncertainties

Suppose  $x$ ,  $y$ ,  $z$  have uncertainties of  $\delta x$ ,  $\delta y$ , and  $\delta z$  and we use those values to calculate some result.

#### Sums & Differences

If

$$f = x + y + z \text{ or } f = x - y - z$$

then

$$df = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

#### Products & Quotients

If

$$f = \frac{x * y}{u * w}$$

then

$$\frac{df}{|f|} = \sqrt{\frac{(dx)^2 + (dy)^2}{(du)^2 + (dw)^2}}$$